MATHEMATICS **STD: X**

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New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 2,3

Simultaneous linear equations

When we think about two linear equations in two variables at the same time, they are called simultaneous equations.

Last year we learnt to solve simultaneous equations by eliminating one variable. Let us revise it.

Ex. (1) Solve the following simultaneous equations.

(1) $5x - 3y = 8;\ 3x + y = 2$

Solution :

Method I : $5x - 3y = 8 \ldots$ (I)

$3x + y = 2 \ldots$ (II)

Multiplying both sides of equation (II) by 3.

$9x + 3y = 6 \ldots$ (III)

$5x - 3y = 8 \ldots$ (I)

Now let us add equations (I) and (III)

$$\begin{aligned} 5x - 3y &= 8 \\ + \quad 9x + 3y &= 6 \\ \hline 14x &= 14 \end{aligned}$$

$\therefore x = 1$

substituting $x = 1$ in equation (II)

$3x + y = 2$

$\therefore 3 \times 1 + y = 2$

$\therefore 3 + y = 2$

$\therefore \quad y = -1$

solution is $x = 1, y = -1$; it is also written as $(x, y) = (1, -1)$

Method (II)

$5x - 3y = 8 \ldots$ (I)

$3x + y = 2 \ldots$ (II)

Let us write value of y in terms of x from equation (II) as

$y = 2 - 3x \quad \ldots$ (III)

Substituting this value of y in equation (I).

$5x - 3y = 8$

$\therefore 5x - 3(2 - 3x) = 8$

$\therefore 5x - 6 + 9x = 8$

$\therefore 14x - 6 = 8$

$\therefore 14x = 8 + 6$

$\therefore 14x = 14$

$\therefore \quad x = 1$

Substituting $x = 1$ in equation (III).

$y = 2 - 3x$

$\therefore y = 2 - 3 \times 1$

$\therefore y = 2 - 3$

$\therefore y = -1$

$x = 1, y = -1$ is the solution.

Ex. (2) Solve : $3x + 2y = 29;\ 5x - y = 18$

Solution : $3x + 2y = 29 \ldots$ (I) and $5x - y = 18 \ldots$ (II)

Let's solve the equations by eliminating 'y'. Fill suitably the boxes below.

Multiplying equation (II) by 2.

$\therefore 5x \times \square - y \times \square = 18 \times \square$

$\therefore 10x - 2y = \square \ldots$ (III)

Let's add equations (I) and (III)

$$\begin{aligned} 3x + 2y &= 29 \\ + \quad \square - \square &= \square \\ \hline \square &= \square \end{aligned}$$

$\therefore x = \square$

Substituting $x = 5$ in equation (I)

$3x + 2y = 29$

$\therefore 3 \times \square + 2y = 29$

$\therefore \square + 2y = 29$

$\therefore \quad 2y = 29 - \square$

$\therefore \quad 2y = \square \qquad \therefore y = \square$

$(x, y) = (\square, \square)$ is the solution.

Ex. (3) Solve : $15x + 17y = 21;\ 17x + 15y = 11$

Solution : $15x + 17y = 21 \ldots$ (I)

$17x + 15y = 11 \ldots$ (II)

In the two equations above, the coefficients of x and y are interchanged. While solving such equations we get two simple equations by adding and subtracting the given equations. After solving these equations, we can easily find the solution.

Let's add the two given equations.

$$\begin{aligned} 15x + 17y &= 21 \\ + \quad 17x + 15y &= 11 \\ \hline 32x + 32y &= 32 \end{aligned}$$

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MATHEMATICS **STD: X** | 2nd **Update** | **Reduced syll. ALGEBRA** | **(SSC MAHARASHTRA BOARD)**

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 4,5

Dividing both sides of the equation by 32.

$x + y = 1 \ . \ . \ . \quad$ (III)

Now, let's subtract equation (II) from (I)

$$\begin{array}{r} 15x + 17y = 21 \\ - \ \underline{17x + 15y = 11} \\ -2x + 2y = 10 \end{array}$$

dividing the equation by 2.

$-x + y = 5 \ . \ . \ . \quad$ (IV)

Now let's add equations (III) and (V).

$$\begin{array}{r} x + y = 1 \\ + \ \underline{-x + y = 5} \\ \therefore \ 2y = 6 \end{array} \qquad \therefore \ y = 3$$

Place this value in equation (III).

$x + y = 1$

$\therefore \ x + 3 = 1$

$\therefore \ x = 1 - 3 \qquad \therefore \ x = -2$

$(x, y) = (-2, 3)$ is the solution.

Practice Set 1.1

(1) Complete the following activity to solve the simultaneous equations.

$5x + 3y = 9$ -----(I)

$2x + 3y = 12$ ------ (II)

Let's add eqations (I) and (II).

$$\begin{array}{r} 5x + 3y = 9 \\ + \ \underline{2x - 3y = 12} \\ \square \ x = \square \end{array}$$

$x = \dfrac{\square}{\square} \qquad x = \square$

Place $x = 3$ in equation (I).

$5 \times \square + 3y = 9$

$3y = 9 - \square$

$3y = \square$

$y = \dfrac{\square}{3}$

$y = \square$

$\therefore$ Solution is $(x, y) = (\square, \square)$.

2. Solve the following simultaneous equations.

~~(1) $3a + 5b = 26; \ a + 5b = 22$~~ ~~(2) $x + 7y = 10; \ 3x - 2y = 7$~~

~~(3) $2x - 3y = 9; \ 2x + y = 13$~~ ~~(4) $5m - 3n = 19; \ m - 6n = -7$~~

~~(5) $5x + 2y = -3; \ x + 5y = 4$~~ ~~(6) $\frac{1}{3}x + y = \frac{10}{3}; \ 2x + \frac{1}{4}y = \frac{11}{4}$~~

(7) $99x + 101y = 499; \ 101x + 99y = 501$

(8) $49x - 57y = 172; \ 57x - 49y = 252$

Let's recall.

Graph of a linear equation in two variables

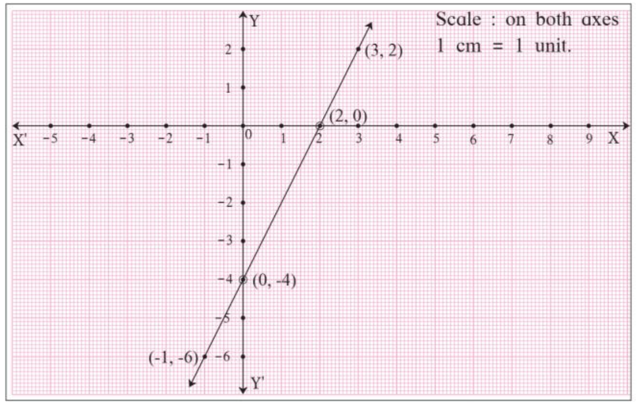
In the 9th standard we learnt that the graph of a linear equation in two variables is a straight line. The ordered pair which satisfies the equation is a solution of that equation. The ordered pair represents a point on that line.

Ex. Draw graph of $2x - y = 4$.

Solution : To draw a graph of the equation let's write 4 ordered pairs.

x	0	2	3	-1
y	-4	0	2	-6
(x, y)	$(0, -4)$	$(2, 0)$	$(3, 2)$	$(-1, -6)$

To obtain ordered pair by simple way let's take $x = 0$ and then $y = 0$.



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New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 16

Let's think.

- What is the nature of solution if D = 0 ?
- What can you say about lines if common solution is not possible?

Practice Set 1.3

1. Fill in the blanks with correct number

$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \square - \square \times 4 = \square - 8 = \square$

2. Find the values of following determinants.

(1) $\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$ (2) $\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$ (3) $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$

3. Solve the following simultaneous equations using Cramer's rule.

(1) $3x - 4y = 10$; $4x + 3y = 5$ (2) $4x + 3y - 4 = 0$; $6x = 8 - 5y$

(3) $x + 2y = -1$; $2x - 3y = 12$ (4) $6x - 4y = -12$; $8x - 3y = -2$

(5) $4m + 6n = 54$; $3m + 2n = 28$ (6) $2x + 3y = 2$; $x - \frac{y}{2} = \frac{1}{2}$

Let's learn.

Equations reducible to a pair of linear equations in two variables

Activity : Complete the following table.

Equation	No. of variables	whether linear or not
$\frac{3}{x} - \frac{4}{y} = 8$	2	Not linear
$\frac{6}{x-1} + \frac{3}{y-2} = 0$	$\square$	$\square$
$\frac{7}{2x+1} + \frac{13}{y+2} = 0$	$\square$	$\square$
$\frac{14}{x+y} + \frac{3}{x-y} = 5$	$\square$	$\square$

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New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 17,18

Let's think.

In the above table the equations are not linear. Can you convert the equations into linear equations ?

Let's remember!

We can create new variables making a proper change in the given variables. Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

Also remember that the denominator of any fraction of the form $\frac{m}{n}$ cannot be zero.

Solved examples

Solve:

Ex. (1) $\frac{4}{x} + \frac{5}{y} = 7;\ \frac{3}{x} + \frac{4}{y} = 5$

Solution : $\frac{4}{x} + \frac{5}{y} = 7;\ \frac{3}{x} + \frac{4}{y} = 5$

$4\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 7 \ldots$ (I)

$3\left(\frac{1}{x}\right) + 4\left(\frac{1}{y}\right) = 5 \ldots$ (II)

Replacing $\left(\frac{1}{x}\right)$ by m and $\left(\frac{1}{y}\right)$ by n in equations (I) and (II), we get

$4m + 5n = 7 \ldots$ (III)

$3m + 4n = 5 \ldots$ (IV)

On solving these equations we get

$m = 3,\ n = -1$

Now, $m = \frac{1}{x}$ $\therefore 3 = \frac{1}{x}$ $\therefore x = \frac{1}{3}$

$n = \frac{1}{y}$ $\therefore -1 = \frac{1}{y}$ $\therefore y = -1$

$\therefore$ Solution of given simultaneous equations is $(x, y) = \left(\frac{1}{3}, -1\right)$

Ex.(2) $\frac{4}{x-y} + \frac{1}{x+y} = 3;\ \frac{2}{x-y} - \frac{3}{x+y} = 5$

Solution : $\frac{4}{x-y} + \frac{1}{x+y} = 3;\ \frac{2}{x-y} - \frac{3}{x+y} = 5$

$4\left(\frac{1}{x-y}\right) + 1\left(\frac{1}{x+y}\right) = 3 \ldots$ (I)

$2\left(\frac{1}{x-y}\right) - 3\left(\frac{1}{x+y}\right) = 5 \ldots$ (II)

Replacing $\left(\frac{1}{x-y}\right)$ by a and $\left(\frac{1}{x+y}\right)$ by b we get

$4a + b = 3 \ldots$ (III)

$2a - 3b = 5 \ldots$ (IV)

On solving these equations we get, $a = 1\ b = -1$

But $a = \left(\frac{1}{x-y}\right)$, $b = \left(\frac{1}{x+y}\right)$

$\therefore \left(\frac{1}{x-y}\right) = 1,\ \left(\frac{1}{x+y}\right) = -1$

$\therefore x - y = 1 \ldots$ (V)

$x + y = -1 \ldots$ (VI)

Solving equation (V) and (VI) we get $x = 0,\ y = -1$

$\therefore$ Solution of the given equations is $(x, y) = (0, -1)$

Let's think.

In the above examples the simultaneous equations obtained by transformation are solved by elimination method.

If you solve these equations by graphical method and by Cramer's rule will you get the same answers ? Solve and check it.

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MATHEMATICS STD: X

2nd Update

Reduced syll. ALGEBRA

(SSC MAHARASHTRA BOARD)

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 19

Activity : To solve given equations fill the boxes below suitably.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \; ; \; \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Replacing $\left(\frac{1}{x-1}\right)$ by m, $\left(\frac{1}{y-2}\right)$ by n

New equations

[ ] ; $6m - 3n = 1$

On solving

$m = \square$, $n = \square$

Replacing m, n by their original values.

$\left(\frac{1}{x-1}\right) = \frac{1}{3}$; [ ]

On solving

$x = \square$, $y = \square$

$\therefore$ $(x, y) = (\; , \;)$ is the solution of the given simultaneous equations.

Practice Set 1.4

1. Solve the following simultaneous equations.

(1) $\frac{2}{x} - \frac{3}{y} = 15$; $\frac{8}{x} + \frac{5}{y} = 77$

(2) $\frac{10}{x+y} + \frac{2}{x-y} = 4$; $\frac{15}{x+y} - \frac{5}{x-y} = -2$

(3) $\frac{27}{x-2} + \frac{31}{y+3} = 85$; $\frac{31}{x-2} + \frac{27}{y+3} = 89$

(4) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$; $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

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MATHEMATICS **STD: X**

2nd Update Reduced syll. ALGEBRA

(SSC MAHARASHTRA BOARD)

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 37,38

$(\ldots\ldots)(x + \sqrt{2}) = 0$

$(\ldots\ldots) = 0$ or $(x + \sqrt{2}) = 0$

$\therefore x = \square$ or $x = -\sqrt{2}$

$\therefore \square$ and $-\sqrt{2}$ are roots of the equation.

(8)* $3x^2 - 2\sqrt{6}\,x + 2 = 0$ (9) $2m\,(m - 24) = 50$

(10) $25m^2 = 9$ (11) $7m^2 = 21m$ (12) $m^2 - 11 = 0$

Let's learn.

Solution of a quadratic equation by completing the square

Teacher : Is $x^2 + 10x + 2 = 0$ a quadratic equation or not ?

Yogesh : Yes Sir, because it is in the form $ax^2 + bx + c = 0$, maximum index of the variable x is 2 and 'a' is non zero.

Teacher : Can you solve this equation ?

Yogesh : No Sir, because it is not possible to find the factors of 2 whose sum is 10.

Teacher : Right, so we have to use another method to solve such equations. Let us learn the method.

Let us add a suitable term to $x^2 + 10x$ so that the new expression would be a complete square.

If $x^2 + 10x + k = (x + a)^2$

then $x^2 + 10x + k = x^2 + 2ax + a^2$

$\therefore 10 = 2a$ and $k = a^2$

by equating the coefficients for the variable x and constant term

$\therefore a = 5 \quad \therefore k = a^2 = (5)^2 = 25$

$\therefore x^2 + 10x + 2 = (x + 5)^2 - 25 + 2 = (x + 5)^2 - 23$

Now can you solve the equation $x^2 + 10x + 2 = 0$?

Rehana : Yes Sir, left side of the equation is now difference of two squares and we can factorise it.

$(x + 5)^2 - (\sqrt{23})^2 = 0$

$\therefore (x + 5 + \sqrt{23})(x + 5 - \sqrt{23}) = 0$

$\therefore x + 5 + \sqrt{23} = 0$ or $x + 5 - \sqrt{23} = 0$

$\therefore x = -5 - \sqrt{23}$ or $x = -5 + \sqrt{23}$

Hameed : Sir, May I suggest another way ?

$(x + 5)^2 - (\sqrt{23})^2 = 0$

$\therefore (x + 5)^2 = (\sqrt{23})^2$

$\therefore x + 5 = \sqrt{23}$ or $x + 5 = -\sqrt{23}$

$\therefore x = -5 + \sqrt{23}$ or $x = -5 - \sqrt{23}$

Solved Examples

Ex. (1) Solve : $5x^2 - 4x - 3 = 0$

Solution : It is convenient to make coefficient of x^2 as 1 and then convert the equation as the of difference of two squares, so dividing the equation by 5,

we get, $x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

now if $x^2 - \frac{4}{5}x + k = (x - a)^2$ then $x^2 - \frac{4}{5}x + k = x^2 - 2ax + a^2$.

compare the terms in $x^2 - \frac{4}{5}x$ and $x^2 - 2ax$.

$-2ax = -\frac{4}{5}x \quad \therefore a = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$

$\therefore k = a^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

Now, $x^2 - \frac{4}{5}x - \frac{3}{5} = 0$

$\therefore x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{3}{5} = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{4}{25} + \frac{3}{5}\right) = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{19}{25}\right) = 0$

$\therefore \left(x - \frac{2}{5}\right)^2 = \left(\frac{19}{25}\right)$

$\therefore x - \frac{2}{5} = \frac{\sqrt{19}}{5}$ or $x - \frac{2}{5} = -\frac{\sqrt{19}}{5}$

$\therefore x = \frac{2}{5} + \frac{\sqrt{19}}{5}$ or $x = \frac{2}{5} - \frac{\sqrt{19}}{5}$

$\therefore x = \frac{2+\sqrt{19}}{5}$ or $x = \frac{2-\sqrt{19}}{5}$

$\therefore \frac{2+\sqrt{19}}{5}$ and $\frac{2-\sqrt{19}}{5}$ are roots of the equation.

When equation is in the form $x^2 + bx + c = 0$, it can be written as $x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$ that is, $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 - c$

MATHEMATICS **STD: X**

2nd Update

Reduced syll. ALGEBRA

(SSC MAHARASHTRA BOARD)

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 39,42

Ex. (2) Solve : $x^2 + 8x - 48 = 0$

Method I : Completing the square.

$x^2 + 8x - 48 = 0$

$\therefore x^2 + 8x + 16 - 16 - 48 = 0$

$\therefore (x + 4)^2 - 64 = 0$

$\therefore (x + 4)^2 = 64$

$\therefore x + 4 = 8$ or $x + 4 = -8$

$\therefore x = 4$ or $x = -12$

Method II : Factorisation

$x^2 + 8x - 48 = 0$

$\therefore x^2 + 12x - 4x - 48 = 0$

$\therefore x(x + 12) - 4(x + 12) = 0$

$\therefore (x + 12)(x - 4) = 0$

$\therefore x + 12 = 0$ or $x - 4 = 0$

$\therefore x = -12$ or $x = 4$

Practice Set 2.3

Solve the following quadratic equations by completing the square method.

(1) $x^2 + x - 20 = 0$ (2) $x^2 + 2x - 5 = 0$ (3) $m^2 - 5m = -3$

(4) $9y^2 - 12y + 2 = 0$ (5) $2y^2 + 9y + 10 = 0$ (6) $5x^2 = 4x + 7$

Let's learn.

Formula for solving a quadratic equation

$ax^2 + bx + c$, Divide the polynomial by a ($\because a \neq 0$) to get $x^2 + \frac{b}{a}x + \frac{c}{a}$.

Let us write the polynomial $x^2 + \frac{b}{a}x + \frac{c}{a}$ in the form of difference of two square numbers. Now we can obtain roots or solutions of equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ which is equivalent to $ax^2 + bx + c = 0$.

$ax^2 + bx + c = 0 \ldots$ (I)

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \ldots\ldots$ dividing both sides by a

$\therefore x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$

$\therefore \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

For more information :

Let us understand the solution of equation $x^2 - 2x - 3 = 0$ when solved graphically.

$x^2 - 2x - 3 = 0 \quad \therefore \quad x^2 = 2x + 3$ The values which satisfy the equation are the roots of the equation.

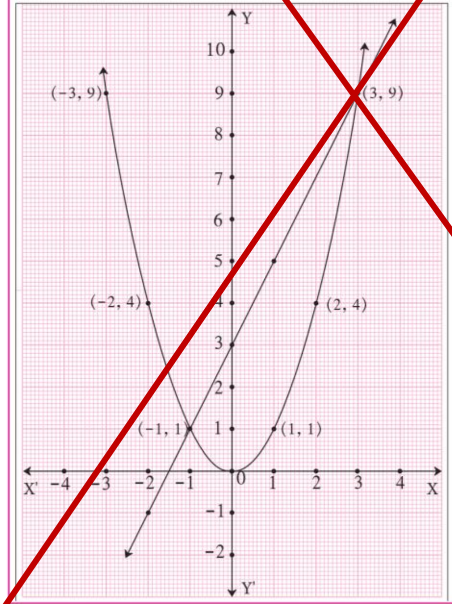
Let $y = x^2 = 2x + 3$. Let us draw graph of $y = x^2$ and $y = 2x + 3$

$y = x^2$

x	3	2	1	0	−1	−2	−3
y	9	4	1	0	1	4	9

$y = 2x + 3$

x	−1	0	1	−2
y	1	3	5	−1



These graphs intersect each other at (−1, 1) and (3, 9).

$\therefore$ The solutions of $x^2 = 2x + 3$ i.e $x^2 - 2x - 3 = 0$ are $x = -1$ or $x = 3$.

In the adjacent diagram the graphs of equations $y = x^2$ and $y = 2x + 3$ are given. From their points of intersection, observe and understand how you get the solutions of $x^2 = 2x + 3$ i.e solutions of $x^2 - 2x - 3 = 0$.

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 43

Ex. (4) $25x^2 + 30x + 9 = 0$

Solution : $25x^2 + 30x + 9 = 0$ comparing the equation with $ax^2 + bx + c = 0$

we get $a = 25,\ b = 30,\ c = 9,$

$\therefore b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900 \quad = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-30 \pm \sqrt{0}}{2 \times 25}$$

$$\therefore \quad x = \frac{-30 + 0}{50} \text{ or } x = \frac{-30 - 0}{50}$$

$$\therefore \quad x = -\frac{30}{50} \text{ or } x = -\frac{30}{50}$$

$$\therefore \quad x = -\frac{3}{5} \text{ or } x = -\frac{3}{5}$$

that is both the roots are equal.

Also note that $25x^2 + 30x + 9 = 0$ means $(5x + 3)^2 = 0$

Ex. (5) $x^2 + x + 5 = 0$

Solution : $x^2 + x + 5 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1,\ b = 1,\ c = 5,$

$\therefore \quad b^2 - 4ac = (1)^2 - 4 \times 1 \times 5$

$= 1 - 20$

$= -19$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{-19}}{2 \times 1}$$

$$= \frac{-1 \pm \sqrt{-19}}{2}$$

But $\sqrt{-19}$ is not a real number. Hence roots of the equation are not real.

Activity : Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, ~~by completing the square method~~ and by using the formula. Verify that you will get the same roots every time.

Practice Set 2.4

1. Compare the given quadratic equations to the general form and write values of a, b, c.

(1) $x^2 - 7x + 5 = 0$ (2) $2m^2 = 5m - 5$ (3) $y^2 = 7y$

2. Solve using formula.

(1) $x^2 + 6x + 5 = 0$ (2) $x^2 - 3x - 2 = 0$ (3) $3m^2 + 2m - 7 = 0$

(4) $5m^2 - 4m - 2 = 0$ (5) $y^2 + \frac{1}{3}y = 2$ (6) $5x^2 + 13x + 8 = 0$

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MATHEMATICS **STD: X**

2nd Update

Reduced syll. ALGEBRA

(SSC MAHARASHTRA BOARD)

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 46,47

Let's learn.

The relation between roots of the quadratic equation and coefficients

α and β are the roots of the equation $ax^2 + bx + c = 0$ then,

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a}$$
$$= -\frac{2b}{2a}$$
$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\alpha \times \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$= \frac{\left(-b+\sqrt{b^2-4ac}\right)\times\left(-b-\sqrt{b^2-4ac}\right)}{4a^2}$$
$$= \frac{b^2-\left(b^2-4ac\right)}{4a^2}$$
$$= \frac{4ac}{4a^2}$$
$$= \frac{c}{a}$$
$$\therefore \alpha\beta = \frac{c}{a}$$

Activity : Fill in the empty boxes below properly
For $10x^2 + 10x + 1 = 0$,
α + β = ☐ and α × β = ☐

Solved examples

Ex. (1) If α and β are the roots of the quadratic equation $2x^2 + 6x - 5 = 0$, then find (α + β) and α × β.

Solution : Comparing $2x^2 + 6x - 5 = 0$ with $ax^2 + bx + c = 0$.

$\therefore a = 2,\ b = 6,\ c = -5$

$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{6}{2} = -3$

and $\alpha \times \beta = \frac{c}{a} = \frac{-5}{2}$

Ex. (2) The difference between the roots of the equation $x^2 - 13x + k = 0$ is 7 find k.

Solution : Comparing $x^2 - 13x + k = 0$ with $ax^2 + bx + c = 0$

$a = 1,\ b = -13,\ c = k$

Let α and β be the roots of the equation.

$\alpha + \beta = -\frac{b}{a} = -\frac{(-13)}{1} = 13$ . . . (I)

But $\alpha - \beta = 7$ . . . . (given) (II)

$2\alpha = 20$ . . . (adding (I) and (II))

$\therefore \alpha = 10$

$\therefore 10 + \beta = 13$ . . . (from (I))

$\therefore \beta = 13 - 10$

$\therefore \beta = 3$

But $\alpha \times \beta = \frac{c}{a}$

$\therefore 10 \times 3 = \frac{k}{1}$

$\therefore k = 30$

Ex. (3) If α and β are the roots of $x^2 + 5x - 1 = 0$ then find -

(i) $\alpha^3 + \beta^3$ (ii) $\alpha^2 + \beta^2$.

Solution : $x^2 + 5x - 1 = 0$

$a = 1,\ b = 5,\ c = -1$

$\alpha + \beta = -\frac{b}{a} = \frac{-5}{1} = -5$

$\alpha \times \beta = \frac{c}{a} = \frac{-1}{1} = -1$

(i) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$= (-5)^3 - 3 \times (-1) \times (-5)$
$= -125 - 15$
$\alpha^3 + \beta^3 = -140$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (-5)^2 - 2 \times (-1)$
$= 25 + 2$
$\alpha^2 + \beta^2 = 27$

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MATHEMATICS **STD: X**

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(SSC MAHARASHTRA BOARD)

New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 49,50

Let's remember!

(1) If α and β are roots of quadratic equation $ax^2 + bx + c = 0$,

(i) $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

(ii) $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$

(2) Nature of roots of quadratic equation $ax^2 + bx + c = 0$ depends on the value of $b^2 - 4ac$. Hence $b^2 - 4ac$ is called discriminant and is denoted by Greek letter Δ.

(3) If $\Delta = 0$ The roots of quadratic equation are real and equal.

If $\Delta > 0$ then the roots of quadratic equation are real and unequal.

If $\Delta < 0$ then the roots of quadratic equation are not real.

(4) The quadratic equation, whose roots are α and β is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Practice Set 2.5

1. Activity : Fill in the gaps and complete.

(1) Quadratic equation $ax^2 + bx + c = 0$ → $b^2 - 4ac = 5$ / $b^2 - 4ac = -5$ → Nature of roots

(2) Sum of roots = -7 → Quadratic equation . . . . ← Product of roots = 5

(3) If α, β are roots of quadratic equation, $2x^2 - 4x - 3 = 0$ → $\alpha + \beta = ...$; $\alpha \times \beta = ...$

2. Find the value of discriminant.

(1) $x^2 + 7x - 1 = 0$ (2) $2y^2 - 5y + 10 = 0$ (3) $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

3. Determine the nature of roots of the following quadratic equations.

(1) $x^2 - 4x + 4 = 0$ (2) $2y^2 - 7y + 2 = 0$ (3) $m^2 + 2m + 9 = 0$

4. Form the quadratic equation from the roots given below.

(1) 0 and 4 (2) 3 and -10 (3) $\frac{1}{2}, -\frac{1}{2}$ (4) $2-\sqrt{5}, 2+\sqrt{5}$

5.★ Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$

6.★ α, β are roots of $y^2 - 2y - 7 = 0$ find,

(1) $\alpha^2 + \beta^2$ (2) $\alpha^3 + \beta^3$

7. The roots of each of the following quadratic equations are real and equal, find k.

(1) $3y^2 + ky + 12 = 0$ (2) $kx(x - 2) + 6 = 0$

Let's learn.

Application of quadratic equation

Quadratic equations are useful in daily life for finding solutions of some practical problems. We are now going to learn the same.

Ex. (1) There is a rectangular onion storehouse in the farm of Mr. Ratnakarrao at Tivasa. The length of rectangular base is more than its breadth by 7 m and diagonal is more than length by 1 m. Find length and breadth of the storehouse.

Solution : Let breadth of the storehouse be x m.

$\therefore$ length $= (x + 7)$ m, diagonal $= x + 7 + 1 = (x + 8)$ m

By Pythagorus theorem

$x^2 + (x + 7)^2 = (x + 8)^2$

$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$

$\therefore x^2 + 14x - 16x + 49 - 64 = 0$

$\therefore x^2 - 2x - 15 = 0$

$\therefore x^2 - 5x + 3x - 15 = 0$

$\therefore x(x - 5) + 3(x - 5) = 0$

$\therefore (x - 5)(x + 3) = 0$

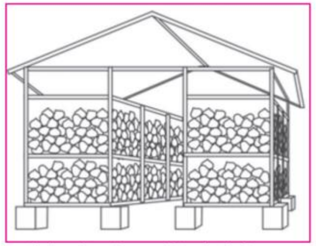
$\therefore x - 5 = 0$ or $x + 3 = 0$

$\therefore x = 5$ or $x = -3$

But length is never negative $\therefore x \neq -3$

$\therefore x = 5$ and $x + 7 = 5 + 7 = 12$

$\therefore$ Length of the base of storehouse is 12m and breadth is 5m.


Onion Storehouse (Kandachal)

MATHEMATICS **STD: X**

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Reduced syll. ALGEBRA

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New Update
Reduced syllabus
class 10
ALGEBRA

Page no. 53,54

Solution : □ABCD is a trapezium.

AB || CD

A (□ABCD) = $\frac{1}{2}$ (AB + CD) × □

33 = $\frac{1}{2}$ (x + 2x + 1) × □

∴ □ = (3x + 1) × □

∴ 3x^2 + □ - □ = 0

∴ 3x (. . . .) +10(. . . .) = 0

∴ (3x +10)(----) = 0

∴ (3x +10) = 0 or □ = 0

∴ x = $-\frac{10}{3}$ or x = □

But length is never negative.

∴ $x \neq -\frac{10}{3}$ ∴ x = □

AB = ---, CD = --- , AD = BC = ---

Problem Set - 2

1. Choose the correct answers for the following questions.

(1) Which one is the quadratic equation ?

(A) $\frac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$ (C) $n - 1 = 2n$ (D) $\frac{1}{x^2}(x + 2) = x$

(2) Out of the following equations which one is not a quadratic equation ?

(A) $x^2 + 4x = 11 + x^2$ (B) $x^2 = 4x$ (C) $5x^2 = 90$ (D) $2x - x^2 = x^2 + 5$

(3) The roots of x^2 + kx + k = 0 are real and equal, find k.

(A) 0 (B) 4 (C) 0 or 4 (D) 2

(4) For $\sqrt{2}x^2 - 5x + \sqrt{2} = 0$ find the value of the discriminant.

(A) -5 (B) 17 (C) $\sqrt{2}$ (D) $2\sqrt{2} - 5$

(5) Which of the following quadratic equations has roots 3, 5 ?

(A) $x^2 - 15x + 8 = 0$ (B) $x^2 - 8x + 15 = 0$

(C) $x^2 + 3x + 5 = 0$ (D) $x^2 + 8x - 15 = 0$

(6) Out of the following equations, find the equation having the sum of its roots -5.

(A) $3x^2 - 15x + 3 = 0$ (B) $x^2 - 5x + 3 = 0$

(C) $x^2 + 3x - 5 = 0$ (D) $3x^2 + 15x + 3 = 0$

(7) $\sqrt{5}m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation ?

(A) Real and uneual roots (B) Real and equal roots

(C) Roots are not real (D) Three roots.

(8) One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

(A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 2

2. Which of the following equations is quadratic ?

(1) $x^2 + 2x + 11 = 0$ (2) $x^2 - 2x + 5 = x^2$ (3) $(x + 2)^2 = 2x^2$

3. Find the value of discriminant for each of the following equations.

(1) $2y^2 - y + 2 = 0$ (2) $5m^2 - m = 0$ (3) $\sqrt{5}x^2 - x - \sqrt{5} = 0$

4. One of the roots of quadratic equation $2x^2$ + kx - 2 = 0 is -2, find k.

5. Two roots of quadratic equations are given ; frame the equation.

(1) 10 and -10 (2) $1-3\sqrt{5}$ and $1+3\sqrt{5}$ (3) 0 and 7

6. Determine the nature of roots for each of the quadratic equation.

(1) $3x^2 - 5x + 7 = 0$ (2) $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$ (3) $m^2 - 2m + 1 = 0$

7. Solve the following quadratic equations.

(1) $\frac{1}{x+5} = \frac{1}{x^2}$ (2) $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$ (3) $(2x + 3)^2 = 25$

(4) $m^2 + 5m + 5 = 0$ (5) $5m^2 + 2m + 1 = 0$ (6) $x^2 - 4x - 3 = 0$

8.★ Find m if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots.

9.★ The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

10.★ Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation $2x^2 + 2(p + q)x + p^2 + q^2 = 0$

11.★ Mukund possesses ₹ 50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

12.★ The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

13.★ Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

14.★ Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meter more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is $\frac{1}{3}$ of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond

15.★ A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely ?

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MATHEMATICS STD: X

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class 10
ALGEBRA

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Page no. 81-112

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MATHEMATICS STD: X

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class 10
ALGEBRA

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Page no. 129-168

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