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This theorem can be proved by indirect method.

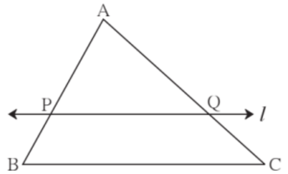

Fig. 1.18

Activity :

- Draw a Δ ABC.
- Bisect $\angle$ B and name the point of intersection of AC and the angle bisector as D.
- Measure the sides.

  AB = ☐ cm   BC = ☐ cm

  AD = ☐ cm   DC = ☐ cm
- Find ratios $\frac{AB}{BC}$ and $\frac{AD}{DC}$.
- You will find that both the ratios are almost equal.
- Bisect remaining angles of the triangle and find the ratios as above. You can verify that the ratios are equal.

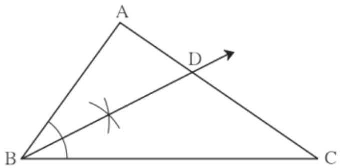

Fig. 1.19

Let's learn.

Property of an angle bisector of a triangle

Theorem : The bisector of an angle of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Given : In Δ ABC, bisector of $\angle$C intersects seg AB in the point E.

To prove : $\frac{AE}{EB} = \frac{CA}{CB}$

Construction : Draw a line parallel to ray CE, passing through the point B. Extend AC so as to intersect it at point D.

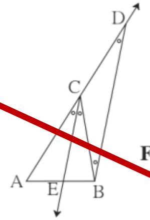

Fig. 1.20

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Proof : ray CE || ray BD and AD is transversal,

$\therefore$ $\angle ACE = \angle CDB$ .... (corresponding angles) ...(I)

Now taking BC as transversal

$\angle ECB = \angle CBD$ .... (alternate angle) ...(II)

But $\angle ACE \cong \angle ECB$ .... (given) ...(III)

$\therefore$ $\angle CBD \cong \angle CDB$ .... [from (I), (II) and (III)]

In Δ CBD, side CB $\cong$ side CD ....(sides opposite to congruent angles)

$\therefore$ CB = CD ...(IV)

Now in Δ ABD, seg EC || seg BD .... (construction)

$\therefore$ $\frac{AE}{EB} = \frac{AC}{CD}$ ....(Basic proportionality theorem)..(V)

$\therefore$ $\frac{AE}{EB} = \frac{AC}{CB}$ .... [from (IV) and (V)]

For more information :

Write another proof of the theorem yourself.

Draw DM $\perp$ AB and DN $\perp$ AC. Use the following properties and write the proof.

(1) The areas of two triangles of equal heights are proportional to their bases.

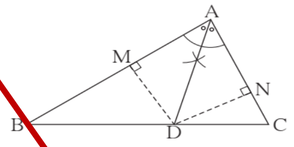

Fig. 1.21

(2) Every point on the bisector of an angle is equidistant from the sides of the angle.

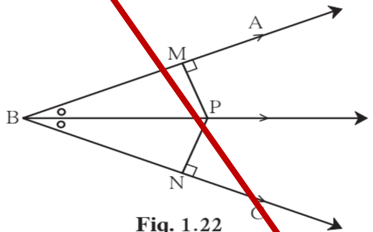

Fig. 1.22

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Converse of angle bisector theorem

If in Δ ABC, point D on side BC such that $\frac{AB}{AC} = \frac{BD}{DC}$, then ray AD bisects $\angle$ BAC.

Property of three parallel lines and their transversals

Activity:

- Draw three parallel lines.
- Label them as l, m, n.
- Draw transversals t_1 and t_2.
- AB and BC are intercepts on transversal t_1.
- PQ and QR are intercepts on transversal t_2.

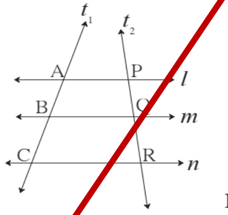

Fig. 1.23

- Find ratios $\frac{AB}{BC}$ and $\frac{PQ}{QR}$. You will find that they are almost equal.

Theorem : The ratio of the intercepts made on a transversal by three parallel lines is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

Given : line l || line m || line n

t_1 and t_2 are transversals.
Transversal t_1 intersects the lines in points A, B, C and t_2 intersects the lines in points P, Q, R.

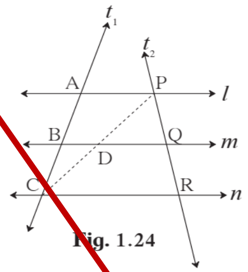

Fig. 1.24

To prove : $\frac{AB}{BC} = \frac{PQ}{QR}$

Proof : Draw seg PC, which intersects line m at point D.

In Δ ACP, BD || AP

$\therefore \frac{AB}{BC} = \frac{PD}{DC}$ . . . . . (I) (Basic proportionality theorem)

In Δ CPR, DQ || CR

$\therefore \frac{PD}{DC} = \frac{PQ}{QR}$ . . . . . (II) (Basic proportionality theorem)

$\therefore \frac{AB}{BC} = \frac{PD}{DC} = \frac{PQ}{QR}$ . . . . . from (I) and (II). $\therefore \frac{AB}{BC} = \frac{PQ}{QR}$

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Remember this!

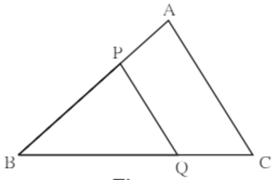

Fig. 1.25

(1) Basic proportionality theorem.

In Δ ABC, if seg PQ $\parallel$ seg AC

then $\frac{AP}{BP} = \frac{QC}{BQ}$

(2) Converse of basic proportionality theorem.

In ΔPQR, if $\frac{PS}{SQ} = \frac{PT}{TR}$

then seg ST $\parallel$ seg QR.

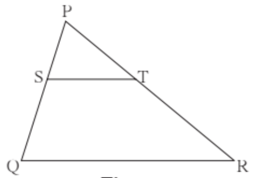

Fig. 1.26

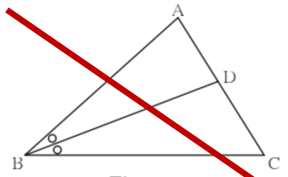

Fig. 1.27

(3) Theorem of bisector of an angle of a triangle.

If in Δ ABC, BD is bisector of $\angle$ ABC,

then $\frac{AB}{BC} = \frac{AD}{DC}$

(4) Property of three parallel lines and their transversals.

If line AX $\parallel$ line BY $\parallel$ line CZ and line l and line m are their transversals then $\frac{AB}{BC} = \frac{XY}{YZ}$

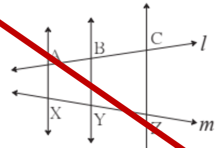

Fig. 1.28

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Solved Examples

Ex. (1) In Δ ABC, DE || BC

If DB = 5.4 cm, AD = 1.8 cm

EC = 7.2 cm then find AE.

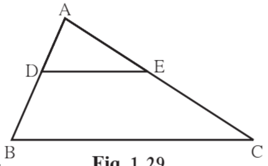

Fig. 1.29

Solution : In Δ ABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ ..... Basic proportionality theorem

$\therefore \frac{1.8}{5.4} = \frac{AE}{7.2}$

$\therefore$ AE × 5.4 = 1.8 × 7.2

$\therefore$ AE = $\frac{1.8 \times 7.2}{5.4}$ = 2.4

AE = 2.4 cm

Ex. (2) In Δ PQR, seg RS bisects ∠ R.

If PR = 15, RQ = 20 PS = 12

then find SQ.

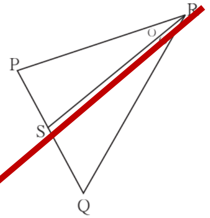

Fig. 1.30

Solution : In Δ PRQ, seg RS bisects ∠ R.

$\frac{PR}{RQ} = \frac{PS}{SQ}$ . . . . property of angle bisector

$\frac{15}{20} = \frac{12}{SQ}$

SQ = $\frac{12 \times 20}{15}$

$\therefore$ SQ = 16

Activity :

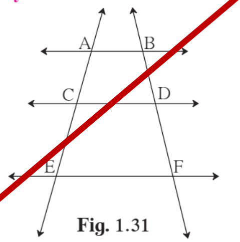

Fig. 1.31

In the figure 1.31, AB || CD || EF

If AC = 5.4, CE = 9, BD = 7.5

then find DF

Solution : AB || CD || EF

$\frac{AC}{\square} = \frac{\square}{DF}$ . . . . . ( )

$\frac{5.4}{9} = \frac{\square}{DF}$ $\therefore$ DF = $\square$

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Activity :

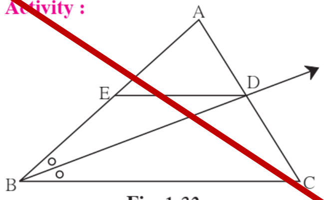

Fig. 1.32

In ΔABC, ray BD bisects $\angle ABC$. A-D-C, side DE || side BC, A-E-B then prove that, $\frac{AB}{BC} = \frac{AE}{EB}$

Proof : In Δ ABC, ray BD bisects $\angle$ B.

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$ . . . (I)(Angle bisector theorem)

In Δ ABC, DE || BC

$\frac{AE}{EB} = \frac{AD}{DC}$ . . . . . (II) (. . . . .)

$\frac{AB}{\square} = \frac{\square}{EB}$ . . . . . . from (I) and (II)

Practice set 1.2

1. Given below are some triangles and lengths of line segments. Identify in which figures, ray PM is the bisector of $\angle$ QPR.

(1)

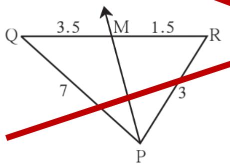

Fig. 1.33

(2)

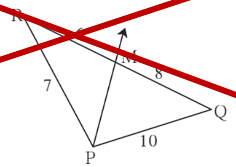

Fig. 1.34

(3)

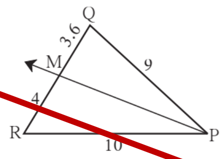

Fig. 1.35

2. In Δ PQR, PM = 15, PQ = 25 PR = 20, NR = 8. State whether line NM is parallel to side RQ. Give reason.

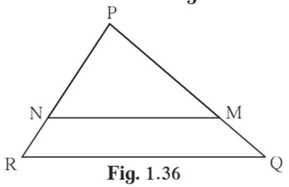

Fig. 1.36

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3. In Δ MNP, NQ is a bisector of ∠N. If MN = 5, PN = 7, MQ = 2.5 then find QP .

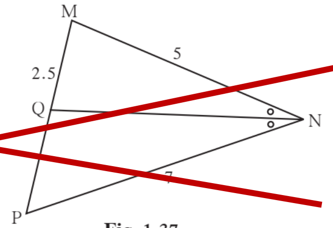

Fig. 1.37

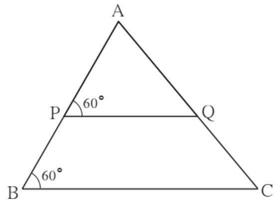

Fig. 1.38

4. Measures of some angles in the figure are given. Prove that

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

5. In trapezium ABCD, side AB || side PQ || side DC, AP = 15, PD = 12, QC = 14, find BQ.

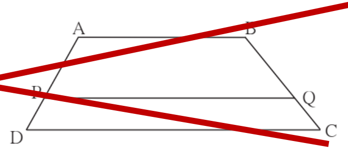

Fig. 1.39

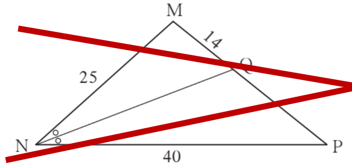

Fig. 1.40

6. Find QP using given information in the figure.

7. In figure 1.41, if AB || CD || FE then find x and AE.

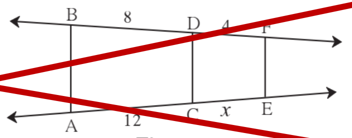

Fig. 1.41

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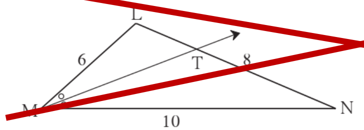

Fig. 1.42

8. In Δ LMN, ray MT bisects $\angle$ LMN. If LM = 6, MN = 10, TN = 8, then find LT.

9. In Δ ABC, seg BD bisects $\angle$ ABC. If AB = x, BC = $x + 5$, AD = $x - 2$, DC = $x + 2$, then find the value of x.

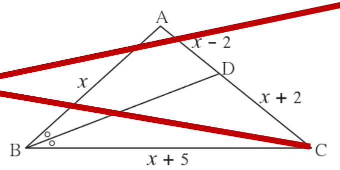

Fig. 1.43

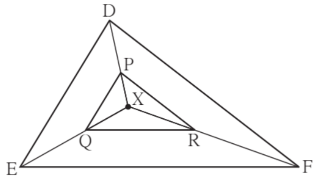

Fig. 1.44

10. In the figure 1.44, X is any point in the interior of triangle. Point X is joined to vertices of triangle. Seg PQ || seg DE, seg QR || seg EF. Fill in the blanks to prove that, seg PR || seg DF.

Proof : In Δ XDE, PQ || DE .... $\square$

$\therefore \frac{XP}{\square} = \frac{\square}{QE}$ .... (I) (Basic proportionality theorem)

In Δ XEF, QR || EF .... $\square$

$\therefore \frac{\square}{\square} = \frac{\square}{\square}$ ....(II) $\square$

$\therefore \frac{\square}{\square} = \frac{\square}{\square}$ .... from (I) and (II)

$\therefore$ seg PR || seg DE .... (converse of basic proportionality theorem)

11★ In Δ ABC, ray BD bisects $\angle$ ABC and ray CE bisects $\angle$ ACB. If seg AB $\cong$ seg AC then prove that ED || BC.

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7. In figure 1.75, A–D–C and B–E–C seg DE || side AB If AD = 5, DC = 3, BC = 6.4 then find BE.

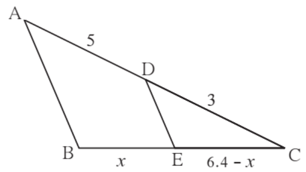

Fig. 1.75

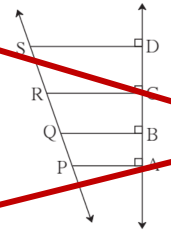

Fig. 1.76

8. In the figure 1.76, seg PA, seg QB, seg RC and seg SD are perpendicular to line AD.

AB = 60, BC = 70, CD = 80, PS = 280 then find PQ, QR and RS.

9. In Δ PQR seg PM is a median. Angle bisectors of ∠PMQ and ∠PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR.

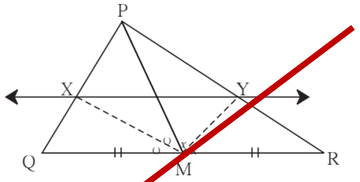

Fig. 1.77

Complete the proof by filling in the boxes.

In Δ PMQ, ray MX is bisector of ∠PMQ.

∴ $\frac{\square}{\square} = \frac{\square}{\square}$ .... (I) theorem of angle bisector.

In Δ PMR, ray MY is bisector of ∠PMR.

∴ $\frac{\square}{\square} = \frac{\square}{\square}$ .... (II) theorem of angle bisector.

But $\frac{MP}{MQ} = \frac{MP}{MR}$ .... M is the midpoint QR, hence MQ = MR.

∴ $\frac{PX}{XQ} = \frac{PY}{YR}$

∴ XY || QR .... converse of basic proportionality theorem.

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10. In fig 1.78, bisectors of ∠ B and ∠ C of Δ ABC intersect each other in point X. Line AX intersects side BC in point Y. AB = 5, AC = 4, BC = 6 then find $\frac{AX}{XY}$.

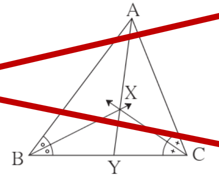

Fig. 1.78

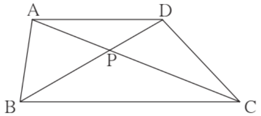

Fig. 1.79

11. In □ ABCD, seg AD || seg BC. Diagonal AC and diagonal BD intersect each other in point P. Then show that $\frac{AP}{PD} = \frac{PC}{BP}$

12. In fig 1.80, XY || seg AC. If 2AX = 3BX and XY = 9. Complete the activity to find the value of AC.

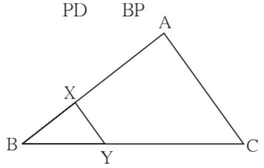

Fig. 1.80

Activity : 2AX = 3BX $\therefore \frac{AX}{BX} = \frac{\square}{\square}$

$\frac{AX+BX}{BX} = \frac{\square + \square}{\square}$ .... by componendo.

$\frac{AB}{BX} = \frac{\square}{\square}$ .... (I)

Δ BCA ~ Δ BYX .... □ test of similarity.

$\therefore \frac{BA}{BX} = \frac{AC}{XY}$ .... corresponding sides of similar triangles.

$\therefore \frac{\square}{\square} = \frac{AC}{9}$ $\therefore$ AC = □ ...from (I)

13*. In figure1.81, the vertices of square DEFG are on the sides of Δ ABC. ∠ A = 90°. Then prove that $DE^2 = BD \times EC$

(Hint : Show that Δ GBD is similar to Δ CFE. Use GD = FE = DE.)

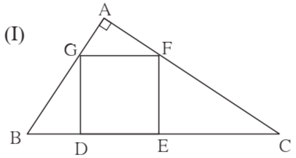

Fig. 1.81

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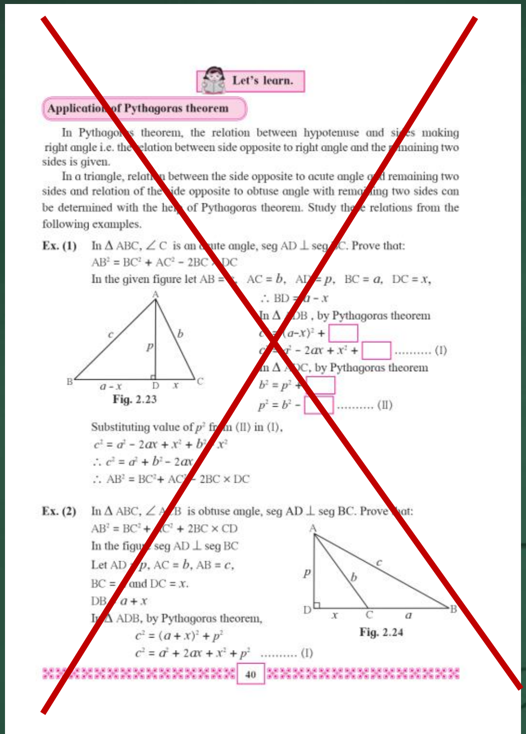

Let's learn.

Application of Pythagoras theorem

In Pythagoras theorem, the relation between hypotenuse and sides making right angle i.e. the relation between side opposite to right angle and the remaining two sides is given.

In a triangle, relation between the side opposite to acute angle and remaining two sides and relation of the side opposite to obtuse angle with remaining two sides can be determined with the help of Pythagoras theorem. Study these relations from the following examples.

Ex. (1) In Δ ABC, $\angle$C is an acute angle, seg AD $\perp$ seg BC. Prove that: $AB^2 = BC^2 + AC^2 - 2BC \times DC$

In the given figure let AB = c, AC = b, AD = p, BC = a, DC = x,

$\therefore$ BD = $a - x$

In Δ ADB, by Pythagoras theorem

$c^2 = (a-x)^2 + \square$

$c^2 = a^2 - 2ax + x^2 + \square$ .... (I)

In Δ ADC, by Pythagoras theorem

$b^2 = p^2 + \square$

$p^2 = b^2 - \square$ .... (II)

Fig. 2.23

Substituting value of p^2 from (II) in (I),

$c^2 = a^2 - 2ax + x^2 + b^2 - x^2$

$\therefore c^2 = a^2 + b^2 - 2ax$

$\therefore AB^2 = BC^2 + AC^2 - 2BC \times DC$

Ex. (2) In Δ ABC, $\angle$ACB is obtuse angle, seg AD $\perp$ seg BC. Prove that: $AB^2 = BC^2 + AC^2 + 2BC \times CD$

In the figure seg AD $\perp$ seg BC

Let AD = p, AC = b, AB = c,

BC = a and DC = x.

DB = $a + x$

In Δ ADB, by Pythagoras theorem,

$c^2 = (a + x)^2 + p^2$

$c^2 = a^2 + 2ax + x^2 + p^2$ .... (I)

Fig. 2.24

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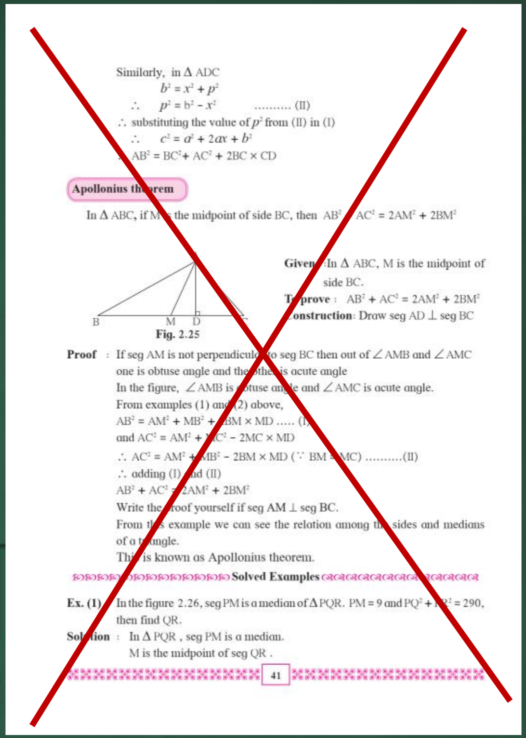

Similarly, in Δ ADC

$b^2 = x^2 + p^2$

$\therefore$ $p^2 = b^2 - x^2$ .... (II)

$\therefore$ substituting the value of p^2 from (II) in (I)

$\therefore$ $c^2 = a^2 + 2ax + b^2$

$\therefore AB^2 = BC^2 + AC^2 + 2BC \times CD$

Apollonius theorem

In Δ ABC, if M is the midpoint of side BC, then $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Given : In Δ ABC, M is the midpoint of side BC.

To prove : $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Construction: Draw seg AD $\perp$ seg BC

Fig. 2.25

Proof : If seg AM is not perpendicular to seg BC then out of $\angle$AMB and $\angle$AMC one is obtuse angle and the other is acute angle

In the figure, $\angle$AMB is obtuse angle and $\angle$AMC is acute angle.

From examples (1) and (2) above,

$AB^2 = AM^2 + MB^2 + 2BM \times MD$ ..... (I)

and $AC^2 = AM^2 + MC^2 - 2MC \times MD$

$\therefore AC^2 = AM^2 + MB^2 - 2BM \times MD$ ($\because$ BM = MC) ....(II)

$\therefore$ adding (I) and (II)

$AB^2 + AC^2 = 2AM^2 + 2BM^2$

Write the proof yourself if seg AM $\perp$ seg BC.

From this example we can see the relation among the sides and medians of a triangle.

This is known as Apollonius theorem.

Solved Examples

Ex. (1) In the figure 2.26, seg PM is a median of Δ PQR. PM = 9 and $PQ^2 + PR^2 = 290$, then find QR.

Solution : In Δ PQR, seg PM is a median.

M is the midpoint of seg QR.

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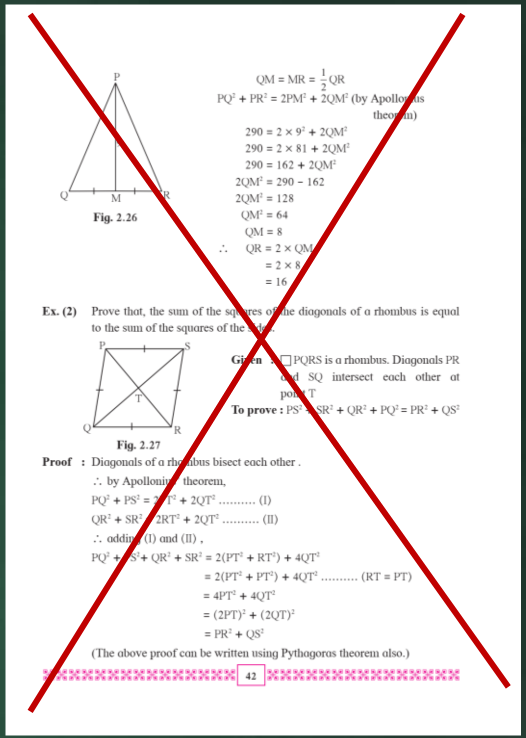

Fig. 2.26

$QM = MR = \frac{1}{2}QR$

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$ (by Apollonius theorem)

$290 = 2 \times 9^2 + 2QM^2$

$290 = 2 \times 81 + 2QM^2$

$290 = 162 + 2QM^2$

$2QM^2 = 290 - 162$

$2QM^2 = 128$

$QM^2 = 64$

$QM = 8$

$\therefore$ $QR = 2 \times QM$

$= 2 \times 8$

$= 16$

Ex. (2) Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.

Given : $\square$PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T

To prove : $PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

Fig. 2.27

Proof : Diagonals of a rhombus bisect each other.

$\therefore$ by Apollonius' theorem,

$PQ^2 + PS^2 = 2PT^2 + 2QT^2$ .... (I)

$QR^2 + SR^2 = 2RT^2 + 2QT^2$ .... (II)

$\therefore$ adding (I) and (II),

$PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + RT^2) + 4QT^2$

$= 2(PT^2 + PT^2) + 4QT^2$ .... (RT = PT)

$= 4PT^2 + 4QT^2$

$= (2PT)^2 + (2QT)^2$

$= PR^2 + QS^2$

(The above proof can be written using Pythagoras theorem also.)

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Practice set 2.2

1. In Δ PQR, point S is the midpoint of side QR.If PQ = 11,PR = 17, PS =13, find QR.
2. In Δ ABC, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB
3. In the figure 2.28 seg PS is the median of Δ PQR and PT $\perp$ QR. Prove that,

   (1) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

   ii) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

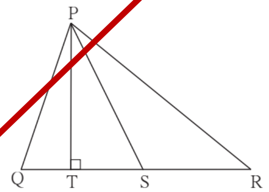

Fig. 2.28

4. In Δ ABC, point M is the midpoint of side BC. If $AB^2 + AC^2 = 290$ cm^2, AM = 8 cm, find BC.

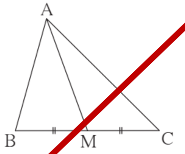

Fig. 2.29

5*. In figure 2.30, point T is in the interior of rectangle PQRS, Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$ (As shown in the figure, draw seg AB || side SR and A-T-B)

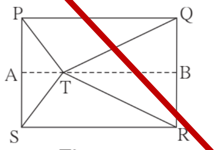

Fig. 2.30

Problem set 2

1. Some questions and their alternative answers are given. Select the correct alternative.

   (1) Out of the following which is the Pythagorean triplet?

   (A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

   (2) In a right angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

   (A) 15 (B) 13 (C) 5 (D) 12

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(3) Out of the dates given below which date constitutes a Pythagorean triplet ?

(A) 15/08/17 (B) 16/08/16 (C) 3/5/17 (D) 4/9/15

(4) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.

(A) Obtuse angled triangle (B) Acute angled triangle

(C) Right angled triangle (D) Equilateral triangle

(5) Find perimeter of a square if its diagonal is $10\sqrt{2}$ cm.

(A) 10 cm (B) $40\sqrt{2}$ cm (C) 20 cm (D) 40 cm

(6) Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

(A) 9 cm (B) 4 cm (C) 6 cm (D) $2\sqrt{6}$ cm

(7) Height and base of a right angled triangle are 24 cm and 18 cm find the length of its hypotenus

(A) 24 cm (B) 30 cm (C) 15 cm (D) 18 cm

(8) In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm. Find measure of $\angle$A.

(A) 30° (B) 60° (C) 90° (D) 45°

2. Solve the following examples.

(1) Find the height of an equilateral triangle having side $2a$.

(2) Do sides 7 cm , 24 cm, 25 cm form a right angled triangle ? Give reason.

(3) Find the length a diagonal of a rectangle having sides 11 cm and 60cm.

(4) Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.

(5) A side of an isosceles right angled triangle is x. Find its hypotenuse.

(6) In Δ PQR; PQ = $\sqrt{8}$, QR = $\sqrt{5}$, PR = $\sqrt{3}$. Is Δ PQR a right angled triangle ? If yes, which angle is of 90° ?

3. In Δ RST, $\angle$S = 90°, $\angle$T = 30°, RT = 12 cm then find RS and ST.

4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

5★. Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

6. In Δ ABC seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$ Find AP.

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7*. Δ ABC is an equilateral triangle. Point P is on base BC such that PC = $\frac{1}{3}$ BC, if AB = 6 cm find AP.

8. From the information given in the figure 2.31, prove that PM = PN = $\sqrt{3} \times a$

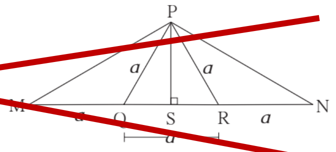

Fig. 2.31

9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was $15\sqrt{2}$ km. Find their speed per hour.

11*. In Δ ABC, $\angle$ BAC = 90°, seg BL and seg CM are medians of Δ ABC. Then prove that: $4(BL^2 + CM^2) = 5\,BC^2$

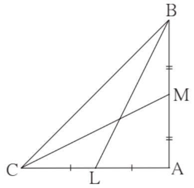

Fig. 2.32

12. Sum of the squares of adjacent sides of a parallelogram is 130 sq cm and length of one of its diagonals is14 cm. Find the length of the other diagonal.

13. In Δ ABC, seg AD $\perp$ seg BC DB = 3CD. Prove that : $2AB^2 = 2AC^2 + BC^2$

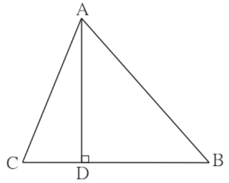

Fig. 2.33

14*. In an isosceles triangle, length of the congruent sides is 13 cm and its base is 10 cm. Find the distance between the vertex opposite the base and the centroid.

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15. In a trapezium ABCD,
seg AB || seg DC
seg BD ⊥ seg AD,
seg AC ⊥ seg BC,
If AD = 15, BC = 15
and AB = 25. Find A(□ ABCD)

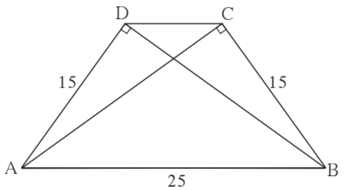

Fig. 2.34

16*. In the figure 2.35, Δ PQR is an equilatral triangle. Point S is on seg QR such that $QS = \frac{1}{3} QR$.
Prove that : $9\ PS^2 = 7\ PQ^2$

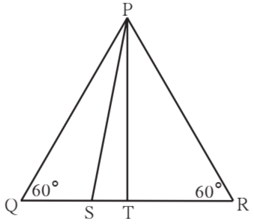

Fig. 2.35

~~**17*.** Seg PM is a median of Δ PQR. If PQ = 40, PR = 42 and PM = 29, find QR.~~
~~**18.** Seg AM is a median of Δ ABC. If AB = 22, AC = 34, BC = 24, find AM.~~

ICT Tools or Links

Obtain information on 'the life of Pythagoras' from the internet. Prepare a slide show.

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You will find that $\angle ACD = \angle ABC$.

You know that $\angle ABC = \frac{1}{2}\ m(\text{arc } AC)$

From this we get $\angle ACD = \frac{1}{2}\ m(\text{arc } AC)$.

Now we will prove this property.

Theorem of angle between tangent and secant

If an angle has its vertex on the circle, its one side touches the circle and the other intersects the circle in one more point, then the measure of the angle is half the measure of its intercepted arc.

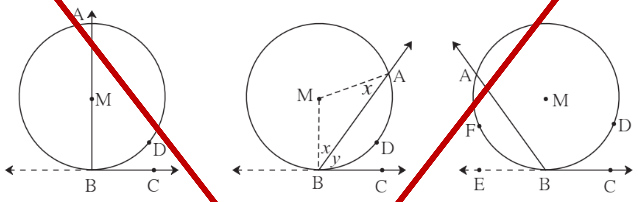

Fig. 3.64

Given : Let $\angle ABC$ be an angle, where vertex B lies on a circle with centre M. Its side BC touches the circle at B and side BA intersects the circle at A. Arc ADB is intercepted by $\angle ABC$.

To prove : $\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

Proof : Consider three cases.

(1) In figure 3.64 (i), the centre M lies on the arm BA of $\angle ABC$,

$\angle ABC = \angle MBC = 90°$ ..... tangent theorem (I)

arc ADB is a semicircle.

$\therefore\ m(\text{arc } ADB) = 180°$ ..... definition of measure of arc (II)

From (I) and (II)

$\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

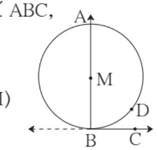

Fig. 3.64(i)

(2) In figure 3.64 (ii) centre M lies in the exterior of $\angle ABC$,

Draw radii MA and MB.

Now, $\angle MBA = \angle MAB$ ..... isosceles triangle theorem

$\angle MBC = 90°$ ..... tangent theorem..... (I)

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Problem set 3

1. Four alternative answers for each of the following questions are given. Choose the correct alternative.

(1) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centers ?

(A) 4.4 cm (B) 8.8 cm (C) 2.2 cm (D) 8.8 or 2.2 cm

(2) Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle ?

(A) 6 cm (B) 12 cm (C) 24 cm (D) can't say

(3) A circle touches all sides of a parallelogram. So the parallelogram must be a, ...

(A) rectangle (B) rhombus (C) square (D) trapezium

(4) Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.

(A) 25 cm (B) 24 cm (C) 7 cm (D) 14 cm

(5) If two circles are touching externally, how many common tangents of them can be drawn?

(A) One (B) Two (C) Three (D) Four

(6) $\angle$ACB is inscribed in arc ACB of a circle with centre O. If $\angle$ACB = 65°, find m(arc ACB).

(A) 65° (B) 130° (C) 295° (D) 230°

~~(7) Chords AB and CD of a circle intersect inside the circle at point E. If AE = 5.6, EB = 10, CE = 8, find ED.~~

~~(A) 7 (B) 8 (C) 11.2 (D) 9~~

(8) In a cyclic □ABCD, twice the measure of $\angle$ A is thrice the measure of $\angle$C. Find the measure of $\angle$C ?

(A) 36 (B) 72 (C) 90 (D) 108

9)* Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. No point, except point B, is common to the arcs. Which is the type of Δ ABC ?

(A) Equilateral triangle (B) Scalene triangle

(C) Right angled triangle (D) Isosceles triangle

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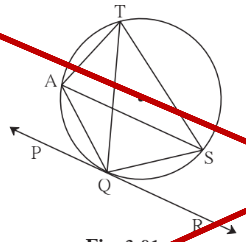

Fig. 3.91

13. In figure 3.91, line PR touches the circle at point Q. Answer the following questions with the help of the figure.

(1) What is the sum of $\angle$TAQ and $\angle$TSQ ?

(2) Find the angles which are congruent to $\angle$AQP.

(3) Which angles are congruent to $\angle$QTS ?

(4) $\angle$TAS = 65°, find the measure of $\angle$TQS and arc TS.

(5) If $\angle$AQP = 42° and $\angle$SQR = 58° find measure of $\angle$ATS.

14. In figure 3.92, O is the centre of a circle, chord PQ $\cong$ chord RS

If $\angle$ POR = 70°

and (arc RS) = 80°, find –

(1) m(arc PR)

(2) m(arc QS)

(3) m(arc QSR)

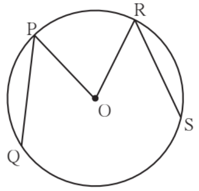

Fig. 3.92

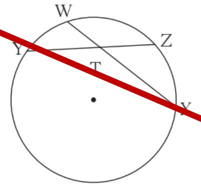

Fig. 3.93

15. In figure 3.93, m(arc WY) = 44°, m(arc ZX) = 68°, then

(1) Find the measure of $\angle$ ZTX.

(2) If WT = 4.8, TX = 8.0, YT = 6.4, find TZ.

(3) If WX = 25, YT = 8, YZ = 26, find WT.

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16. In figure 3.94,

(1) m(arc CE) = 54°,

m(arc BD) = 23°, find measure of ∠CAE.

(2) If AB = 4.2, BC = 5.4,

AE = 12.0, find AD

(3) If AB = 3.6, AC = 9.0,

AD = 5.4, find AE

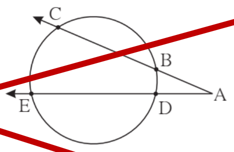

Fig. 3.94

17. In figure 3.95, chord EF || chord GH. Prove that, chord EG ≅ chord FH.

Fill in the blanks and write the proof.

Proof : Draw seg GF.

∠ EFG = ∠ FGH .... ☐ (I)

∠ EFG = ☐ .... inscribed angle theorem (II)

∠ FGH = ☐ .... inscribed angle theorem (III)

∴ m(arc EG) = ☐ from (I), (II), (III).

chord EG ≅ chord FH .... ☐

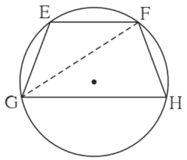

Fig. 3.95

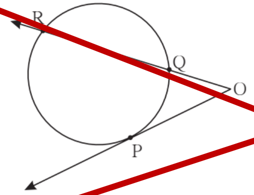

Fig. 3.96

18. In figure 3.96 P is the point of contact.

(1) If m(arc PR) = 140°,

∠ POR = 36°,

find m(arc PQ)

(2) If OP = 7.2, OQ = 3.2,

find OR and QR

(3) If OP = 7.2, OR = 16.2,

find QR.

19. In figure 3.97, circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA ≅ seg AB.

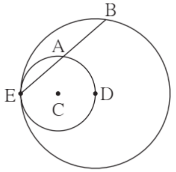

Fig. 3.97

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20. In figure 3.98, seg AB is a diameter of a circle with centre O . The bisector of $\angle$ ACB intersects the circle at point D. Prove that, seg AD $\cong$ seg BD. Complete the following proof by filling in the blanks.

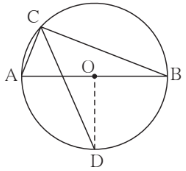

Fig. 3.98

Proof : Draw seg OD.

$\angle$ ACB = ____ .... angle inscribed in semicircle

$\angle$ DCB = ____ .... CD is the bisector of $\angle$ C

m(arc DB) = ____ .... inscribed angle theorem

$\angle$ DOB = ____ .... definition of measure of an arc (I)

seg OA $\cong$ seg OB .... ____ (II)

$\therefore$ line OD is ____ of seg AB .... From (I) and (II)

$\therefore$ seg AD $\cong$ seg BD

21. In figure 3.99, seg MN is a chord of a circle with centre O. MN = 25, L is a point on chord MN such that ML = 9 and d(O,L) = 5. Find the radius of the circle.

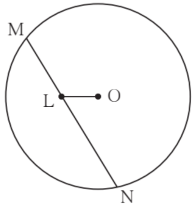

Fig. 3.99

22*. In figure 3.100, two circles intersect each other at points S and R. Their common tangent PQ touches the circle at points P, Q. Prove that, $\angle$ PRQ + $\angle$ PSQ = 180°

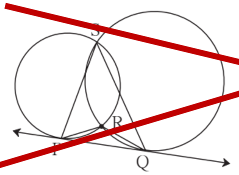

Fig. 3.100

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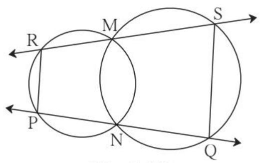

Fig. 3.101

23★. In figure 3.101, two circles intersect at points M and N. Secants drawn through M and N intersect the circles at points R, S and P, Q respectively.
Prove that : seg SQ || seg RP.

24★. In figure 3.102, two circles intersect each other at points A and E. Their common secant through E intersects the circles at points B and D. The tangents of the circles at points B and D intersect each other at point C. Prove that □ ABCD is cyclic.

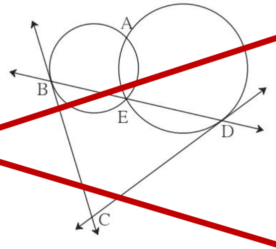

Fig. 3.102

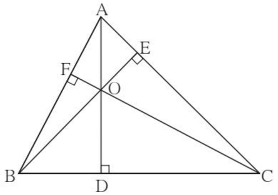

Fig. 3.103

25★. In figure 3.103, seg AD ⊥ side BC, seg BE ⊥ side AC, seg CF ⊥ side AB. Ponit O is the orthocentre. Prove that , point O is the incentre of Δ DEF.

ICT Tools or Links

Use the geogebra to verify the properties of chords and tangents of a circle.

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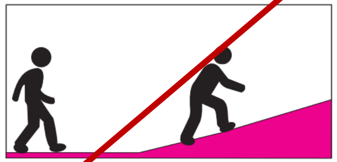
8. In Δ ABC, G (−4, −7) is the centroid. If A (−14, −19) and B(3, 5) then find the co-ordinates of C.
9. A(h, −6), B(2, 3) and C(−6, k) are the co-ordinates of vertices of a triangle whose centroid is G (1, 5). Find h and k.
10. Find the co-ordinates of the points of trisection of the line segment AB with A(2, 7) and B(−4, −8).
11. If A (−14, −10), B(6, −2) is given, find the coordinates of the points which divide segment AB into four equal parts.
12. If A (20, 10), B(0, 20) are given, find the coordinates of the points which divide segment AB into five congruent parts.

Let's learn.

Slope of a line

When we walk on a plane road we need not exert much effort but while climbing up a slope we need more effort. In science, we have studied that while climbing up a slope we have to work against gravitational force.



In co-ordinate geometry, slope of a line is an important concept. We will learn it through the following activity.

Activity I :

In the figure points A(−2, −5), B(0,−2), C(2,1), D(4,4), E(6,7) lie on line l. Observe the table which is made with the help of coordinates of these points on line l.

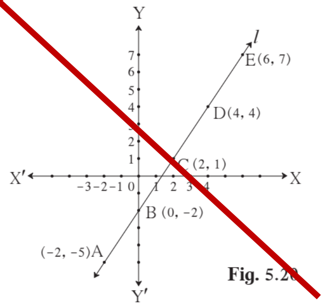


Fig. 5.20

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6. Find k, if R(1, −1), S (−2, k) and slope of line RS is −2.
7. Find k, if B(k, −5), C (1, 2) and slope of the line is 7.
8. Find k, if PQ || RS and P(2, 4), Q (3, 6), R(3, 1), S(5, k).

Problem set 5

1. Fill in the blanks using correct alternatives.

(1) Seg AB is parallel to Y-axis and coordinates of point A are (1,3) then co-ordinates of point B can be ....

(A) (3,1) (B) (5,3) (C) (3,0) (D) (1,−3)

(2) Out of the following, point .... lies to the right of the origin on X- axis.

(A) (−2,0) (B) (0,2) (C) (2,3) (D) (2,0)

(3) Distance of point (−3,4) from the origin is ....

(A) 7 (B) 1 (C) 5 (D) −5

(4) A line makes an angle of 30° with the positive direction of X- axis. So the slope of the line is ....

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

2. Determine whether the given points are collinear.

(1) A(0,2), B(1,−0.5), C(2,−3)

(2) P(1, 2), Q(2, $\frac{8}{5}$), R(3, $\frac{6}{5}$)

(3) L(1,2), M(5,3), N(8,6)

3. Find the coordinates of the midpoint of the line segment joining P(0,6) and Q(12,20).

4. Find the ratio in which the line segment joining the points A(3,8) and B(−9, 3) is divided by the Y- axis.

5. Find the point on X-axis which is equidistant from P(2,−5) and Q(−2,9).

6. Find the distances between the following points.

(i) A(a, 0), B(0, a) (ii) P(−6, −3), Q(−1, 9) (iii) R(−3a, a), S(a, −2a)

7. Find the coordinates of the circumcentre of a triangle whose vertices are (−3,1), (0,−2) and (1,3)

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8. In the following examples, can the segment joining the given points form a triangle ? If triangle is formed, state the type of the triangle considering sides of the triangle.

(1) L(6,4) , M(–5,–3) , N(–6,8)

(2) P(–2,–6) , Q(–4,–2), R(–5,0)

(3) A($\sqrt{2}$, $\sqrt{2}$), B($-\sqrt{2}$, $-\sqrt{2}$), C($-\sqrt{6}$, $\sqrt{6}$)

9. Find k if the line passing through points P(–12, –3) and Q(4, k) has slope $\frac{1}{2}$.

10. Show that the line joining the points A(4, 8) and B(5, 5) is parallel to the line joining the points C(2,4) and D(1,7).

11. Show that points P(1,–2), Q(5,2), R(3,–1), S(–1,–5) are the vertices of a parallelogram

12. Show that the □ PQRS formed by P(2,1), Q(–1,3), R(–5,–3) and S(–2,–5) is a rectangle

13. Find the lengths of the medians of a triangle whose vertices are A(–1, 1), B(5, –3) and C(3, 5) .

14*. Find the coordinates of centroid of the triangles if points D(–7, 6), E(8, 5) and F(2, –2) are the mid points of the sides of that triangle.

15. Show that A(4, –1), B(6, 0), C(7, –2) and D(5, –3)are vertices of a square.

16. Find the coordinates of circumcentre and radius of circumcircle of Δ ABC if A(7, 1), B(3, 5) and C(2, 0) are given.

17. Given A(4,–3), B(8,5). Find the coordinates of the point that divides segment AB in the ratio 3:1.

18*. Find the type of the quadrilateral if points A(–4, –2), B(–3, –7) C(3, –2) and D(2, 3) are joined serially.

19*. The line segment AB is divided into five congruent parts at P, Q, R and S such that A–P–Q–R–S–B. If point Q(12, 14) and S(4, 18) are given find the coordinates of A, P, R,B.

20. Find the coordinates of the centre of the circle passing through the points P(6,–6), Q(3,–7)and R(3,3).

21*. Find the possible pairs of coordinates of the fourth vertex D of the parallelogram, if three of its vertices are A(5,6), B(1,–2)and C(3,–2).

22. Find the slope of the diagonals of a quadrilateral with vertices A(1,7), B(6,3), C(0,–3) and D(–3,3).

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