BOARD QUESTION PAPER: MARCH 2022

MATHEMATICS AND STATISTICS

Time: 3 Hrs **Max. Marks: 80**

Notes:

(i) All questions are compulsory.
(ii) There are 6 questions divided into two sections.
(iii) Write answers of Section-I and Section-II in the same answer book.
(iv) Use of logarithmic tables is allowed. Use of calculator is not allowed.
(v) For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.
(vi) Start answer to each question on a new page.
(vii) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet eg. (a) …………./(b)………../(c)………../ (d)……….. No mark(s) shall be given if "ONLY" the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.

Section – I

Q.1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): (6)[12]

(i) If A = $\begin{bmatrix} 2 & 3 \\ a & 6 \end{bmatrix}$ is a singular matrix, then a = _______.

(a) 6 (b) -5 (c) 3 (d) 4

(ii) $\int \frac{1}{\sqrt{x^2-9}}\,dx$ = _______

(a) $\frac{1}{3}\log\left|x+\sqrt{x^2-9}\right|+c$ (b) $\log\left|x+\sqrt{x^2-9}\right|+c$

(c) $3\log\left|x+\sqrt{x^2-9}\right|+c$ (d) $\log\left|x-\sqrt{x^2-9}\right|+c$

(iii) The slope of a tangent to the curve $y = 3x^2 - x + 1$ at (1, 3) is _______.

(a) 5 (b) -5 (c) $\frac{-1}{5}$ (d) $\frac{1}{5}$

(iv) The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}} = 8\left(\frac{d^3y}{dx^3}\right)$ are respectively _______.

(a) 3, 1 (b) 1, 3 (c) 3, 3 (d) 1, 1

(v) The area of the region bounded by the curve $y = x^2$, $x = 0$, $x = 3$ and X-axis is _______.

(a) 9 sq. units (b) $\frac{26}{3}$ sq. units (c) $\frac{52}{3}$ sq. units (d) 18 sq. units

(vi) $\int_{-5}^{5} \frac{x^7}{x^4+10}\,dx$ = _______

(a) 10 (b) 5 (c) 0 (d) $\frac{1}{5}$

(B) State whether the following statements are true or false (1 mark each): (3)

(i) If f '$(x) > 0$ for all $x \in$ (a, b), then f (x) is decreasing function in the interval (a, b).

(ii) If $\int \frac{4e^x-25}{2e^x-5}\,dx = Ax - 3\log|2e^x-5| + c$, where c is the constant of integration, then A = 5.

(iii) The integrating factor of the differential equation $\frac{dy}{dx}+\frac{y}{x} = x^3$ is $-x$.

(C) Fill in the following blanks (1 mark each): **(3)**

(i) If p ∨ q is true, then the truth value of ~p ∧ ~q is _______.

(ii) $\int \frac{x}{(x+2)(x+3)}\,dx =$ _______ $+ \int \frac{3}{x+3}\,dx$

(iii) $y^2 = (x + c)^3$ is the general solution of the differential equation _______.

Q.2. (A) Attempt any TWO of the following (3 marks each): **(6)[14]**

(i) Write the converse, inverse and contrapositive of the statement, "If 2 + 5 = 10, then 4 + 10 = 20."

(ii) If $x = \sqrt{1+u^2}$, $y = \log(1 + u^2)$, then find $\frac{dy}{dx}$.

(iii) Find the area between the two curves (parabolas) $y^2 = 7x$ and $x^2 = 7y$.

(B) Attempt any TWO of the following (4 marks each): **(8)**

(i) Determine whether the following statement pattern is a tautology, contradiction or contingency:
[(~p ∧ q) ∧ (q ∧ r)] ∧ (~q)

(ii) If $ax^2 + 2hxy + by^2 = 0$, then prove that $\frac{d^2y}{dx^2} = 0$.

(iii) Evaluate: $\int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}}\,dx$.

Q.3. (A) Attempt any TWO of the following questions (3 marks each): **(6)[14]**

(i) Find x, y, z if $\left\{5\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -2 \\ 1 & 3 \end{bmatrix}\right\}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} x+1 \\ y-1 \\ 3z \end{bmatrix}$

(ii) Divide 20 into two parts, so that their product is maximum.

(iii) Solve the following differential equation
$x^2y\,dx - (x^3 + y^3)dy = 0$

(B) Attempt any ONE of the following: **(4)**

(i) Find the inverse of the matrix A by using adjoint method,

where $A = \begin{bmatrix} -3 & -1 & 1 \\ 0 & 0 & 1 \\ -15 & 6 & -6 \end{bmatrix}$

(ii) Evaluate : $\int_1^3 \log x\,dx$.

(C) Attempt any ONE of the following questions (Activity): **(4)**

(i) Complete the following activity to find MPC, MPS, APC and APS, if the expenditure E_c of a person with income I is given as:
$E_c = (0.0003)\,I^2 + (0.075)\,I$
when I = 1000

Solution: Given $E_c = (0.0003)\,I^2 + (0.075)\,I$

we have APC $= \frac{E_c}{I}$

At I = 1000, APC = $\boxed{}$

Now, MPC $= \frac{d(E_c)}{d_I}$

At I = 1000, MPC = $\boxed{}$

At I = 1000, MPS = $\boxed{}$

At I = 1000, APS = $\boxed{}$

(ii) In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, complete the following activity to find the number of times the bacteria are increased in 12 hours.

Solution: Let N be the number of bacteria present at time t.
Since the rate of increase is proportional to the number present.

$\therefore$ $\frac{dN}{dt} = K \boxed{\quad}$; where K is the constant of proportionality.

Integrating on both sides, we get

$\log N = K \boxed{\quad} + C$ …(I)

(i) If $t = 0$ then $N = N_0$
from equation (I);
$\log N_0 = 0 + C$
$\therefore$ $C = \log N_0$

(ii) If $t = 4$ hours then $N = 2\ N_0$; from equation (I);
$K = \boxed{\quad}$

(iii) When $t = 12$ hours
$N = \boxed{\quad} N_0$

Section – II

Q.4. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): **(6)[12]**

(i) The difference between face value and present worth is called _______.
(a) Banker's discount (b) True discount
(c) Banker's gain (d) Cash value

(ii) $b_{xy} . b_{yx} =$ _______.
(a) V(X) (b) σx (c) r^2 (d) σ_y^2

(iii) The assignment problem is said to be balanced, if it is a _______.
(a) square matrix (b) rectangular matrix
(c) row matrix (d) column matrix

(iv) Price index number by weighted aggregate method is given by _______.
(a) $\sum \frac{p_1 w}{p_0 w} \times 100$ (b) $\sum \frac{p_0 w}{p_1 w} \times 100$
(c) $\frac{\sum p_1 w}{\sum p_0 w} \times 100$ (d) $\frac{\sum p_0 w}{\sum p_1 w} \times 100$

(v) The following function represents the p.d.f. of a r.v. X

$$f(x) = \begin{cases} kx ; \text{for } 0 < x < 2 \\ 0 \ ; \text{ otherwise} \end{cases}$$

then the value of K is
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) 0

(vi) If $X \sim B\left(20, \frac{1}{10}\right)$, then E(X) = _______.
(a) 2 (b) 5 (c) 4 (d) 3

(B) State whether the following statements are true or false: (1 mark each) **(3)**

(i) If $X \sim P(m)$ with $P(X = 1) = P(X = 2)$ then $m = 1$.
(ii) Dorbish-Bowley's Price Index Number is square root of product of Laspeyre's and Paasche's Index Numbers.
(iii) To convert maximization type assignment problem into a minimization problem, the smallest element in the matrix is deducted from all elements of matrix.

(C) Fill in the blanks (1 mark each) **(3)**

(i) A wholeseller allows 25% trade discount and 5% cash discount. The net price of an article marked at ₹ 1,600 is _______.

(ii) For a certain bivariate data on 5 pairs of observations given:
$\sum x = 20, \sum y = 20, \sum x^2 = 90, \sum y^2 = 90, \sum xy = 76$ then b_{xy} = ______.

(iii) If P_{01} (L) = 121, P_{01} (P) = 100, then P_{01} (F) = ______.

Q.5. (A) Attempt any TWO of the following question (3 marks each): **(6)[14]**

(i) Find the equation of line of regression of y on x for the following data:
$n = 8, \sum(x_i - \bar{x}).(y_i - \bar{y}) = 120, \bar{x} = 20, \bar{y} = 36, \sigma_x = 2, \sigma_y = 3$

(ii) A job prodcution unit has four jobs P, Q, R, S which can be manufactured on each of the four machines I, II, III and IV. The processing cost of each job for each machine is given in the following table.

Job	Machines (Processing cost in ₹)			
	I	II	III	IV
P	31	25	33	29
Q	25	24	23	21
R	19	21	23	24
S	38	36	34	40

Find the optimal assignment to minimize the total processing cost.

(iii) In a cattle breeding firm, it is prescribed that the food ration for one animal must contain 14, 22 and 1 unit of nutrients A, B and C respectively. Two different kinds of fodder are available. Each unit weihgt of these two contains the following amounts of these three nurients:

Fodder / Nurient	**Fodder 1**	**Fodder 2**
A	2	1
B	2	3
C	1	1

The cost of fodder 1 is ₹ 3 per unit and that of fodder 2 is ₹ 2 per unit. Formulate the L.P.P. to minimize the cost.
[Note: *The question has been modified***]**

(B) Attempt any TWO of the following questions: (4 marks each) **(8)**

(i) Calculate the cost of living index number for the following data by aggregative expenditure method:

Group	**Base Year**		**Current Year**
	Price	**Quantity**	**Price**
Food	120	15	170
Clothing	150	20	190
Fuel and lighting	130	30	220
House rent	160	10	180
Miscellaneous	200	11	220

(ii) Five jobs are performed first on machine M_1 and then on machine M_2. Time taken in hours by each job on each machine is given below:

Jobs→ / Machines ↓	**1**	**2**	**3**	**4**	**5**
M_1	6	8	4	5	7
M_2	3	7	6	4	16

Determine the optimal sequence of jobs and total elapsed time. Also fine the idle time for two machines.

(iii) The probability distribution of a discrete r.v. X is as follows:

x	1	2	3	4	5	6
P(X = x)	K	2k	3k	4k	5k	6k

(a) Determine the value of k.
(b) Fine P $(X \leq 4)$
(c) P $(2 < X < 4)$
(d) $P(X \geq 3)$

Q.6. (A) Attempt any TWO of the following questions (3 marks each) : **(6)[14]**

(i) For 50 students of a class, the regression equation of marks in statistics (X) on the marks in accountancy (Y) is $3y - 5x + 180 = 0$. The variance of marks in statistics is $\left(\frac{9}{16}\right)^{\text{th}}$ of the variance of marks in accountancy. Find the correlation coefficient between marks in two subjects.

(ii) Solve the following L.P.P.
Maximize $z = 13x + 9y$
Subject to $3x + 2y \leq 12$,
$x + y \geq 4$,
$x \geq 0, y \geq 0$.

(iii) Obtain the trend values for the following data using 5 yearly moving averages:

Year	2000	2001	2002	2003	2004
Production xi	10	15	20	25	30
Year	2005	2006	2007	2008	2009
Production xi	35	40	45	50	55

(B) Attempt any ONE of the following questions: **(4)**

(i) A warehouse valued at ₹ 40,000 contains goods worth ₹ 2,40,000. The warehouse is insured against fire for ₹ 16,000 and the goods to the extent of 90% of their value. Goods worth ₹ 80,000 are completely destroyed, while the remaining goods are destroyed to 80% of their value due to fire. The damage to the warehouse is to the extent of ₹ 8,000. Find the total amount that can be claimed under the policy.

(ii) A bill was drawn on 14^{th} April 2005 for ₹ 3,500 and was discounted on 6^{th} July 2005 at 5% p.a. The banker paid ₹ 3,465 for the bill. Find the period of the bill.

(C) Attempt any ONE of the following questions (Activity): **(4)**

(i) An examination consists of 5 multiple choice questions, in each of which the candidate has to decide which one of 4 suggested answers is correct. A completely unprepared student guesses each answer completely randomly. Complete the following activity to find the probability that,
(a) the student gets 4 or more correct answers.
(b) the student gets less than 4 correct answer.

Solution: Let X = No. of correct answers
P = Probability of guessing a correct answer

$\therefore$ $p = \boxed{}$, $q = \boxed{}$

Here n = 5

$\therefore$ $X \sim B(n, p)$

For binomial distribution,

$p(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

(a) Probability that the student gets 4 or more correct answers -
$= P(X \geq 4)$
$= P(X = 4) + P(X = 5)$
$= \boxed{}$

(b) Probability that the student gets less than 4 correct answers -
$= P(X < 4)$
$= 1 - P(X \geq 4)$
$= \boxed{}$

(ii) Following table shows the amount of sugar production (in lakh tonnes) for the year 1931 to 1941:

Year	Production	Year	Production
1931	1	1937	8
1932	0	1938	6
1933	1	1939	5
1934	2	1940	1
1935	3	1941	4
1936	2		

Complete the following activity to fit a trend line by method of least squares:

Solution: Let y_t be the trend line represented by the equation $y_t = a + bt$

Let $u = \dfrac{\text{t - mind value}}{h}$,

mid value = 1936 and h = 1

Year (t)	y_t	u	u^2	uy_t
1931	1	–5	25	–5
1932	0	–4	16	0
1933	1	–3	9	–3
1934	2	–2	4	–4
1935	3	–1	1	–3
1936	2	0	0	0
1937	8	1	1	8
1938	6	2	4	12
1939	5	3	9	15
1940	1	4	16	4
1941	4	5	25	20
	$\sum y_t = 33$	$\sum u = 0$	$\sum u^2 = 110$	$\boxed{}$

The equation of the trend line becomes,
$y_t = a' + b'u$ ....(1)
Two normal equations are,
$\sum y_t = na' + b'\sum u$ ....(2)
$\sum u.y_t = a'\sum u + b'\sum u^2$ ....(3)
From equation (2), we get
$a' = \boxed{}$

From equation (3), we get
$b' = \boxed{}$

The equation of trend line is given by
$y_t = \boxed{}$