BOARD QUESTION PAPER : JULY 2024
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

(i) All questions are compulsory.
(ii) There are 6 questions divided into two sections.
(iii) Write answers of Section-I and Section-II in the same answer book.
(iv) Use of logarithmic tables is allowed. Use of calculator is not allowed.
(v) For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.
(vi) Start answer to each question on a new page.
(vii) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter eg.
(a) ……/(b)……../(c)……../ (d)…….. No mark(s) shall be given if "ONLY" the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.

Section – I

Q.1. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): **[12] (6)**

i. Which of the following sentences is a statement in logic:
(A) He is a good actor.
(B) Did you eat lunch yet?
(C) Every real number is a complex number.
(D) Bring the motor car here.

ii. If $y = 2x^2 + \log 2 + 5$ then $\frac{dy}{dx} =$ _____.
(A) x (B) $4x$ (C) $2x + \log 2$ (D) $-4x$

iii. If $x = 2at^2, y = 4at$ then $\frac{dy}{dx} =$ _____.
(A) $-\frac{1}{2at^2}$ (B) $\frac{1}{2at^3}$ (C) $\frac{1}{t}$ (D) $\frac{1}{4at^3}$

iv. The equation of tangent to the curve $y = x^2 + 4x + 1$ at P(–1, –2) is _____.
(A) $2x - y = 0$ (B) $x + 2y + 5 = 0$
(C) $2x + 4 = 3y$ (D) $5x + y = 1$

v. $\int_{-2}^{3} \frac{dx}{x+5} =$ _____.
(A) $-\log\left(\frac{8}{3}\right)$ (B) $3\log\left(\frac{3}{8}\right)$ (C) $\log\left(\frac{8}{3}\right)$ (D) $-2\log\left(\frac{3}{8}\right)$

vi. The order and degree of the differential equation $\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 + 8 = 0$ are _____.
(A) order = 2, degree = 2 (B) order = 1, degree = 2
(C) order = 1, degree = 1 (D) order = 2, degree = 1

(B) State whether the following statements are true or false (1 mark each): **(3)**

i. Every identity matrix is a scalar matrix.

ii. The rate of change of demand (x) of a commodity w.r.t. its price (y) is $\frac{dy}{dx}$

iii. The integrating factor of $\frac{dy}{dx} - y = x$ is e^x.

(C) Fill in the following blanks (1 mark each): **(3)**

i. If $y = x.\log x$ then $\frac{d^2y}{dx^2}$ _____.

ii. If the marginal revenue $R_m = 40$ and elasticity of demand η is 5, then the average revenue R_A will be _____.

iii. Area of the region bounded by $y = x^4$, $x = 1$, $x = 5$ and the X-axis will be _____.

Q.2. (A) Attempt any TWO of the following questions (3 marks each): **(6) [14]**

i. Write converse, inverse and contrapositive of the following statement:
If the train reaches on time then I catch the connecting flight.

ii. If $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & a \\ -1 & b \end{bmatrix}$ and if $(A + B)^2 = A^2 + B^2$, find value of a and b.

iii. Find $\frac{dy}{dx}$, if $y = (x)^x + (a)^x$

(B) Attempt any TWO of the following questions (4 marks each): **(8)**

i. Evaluate: $\int \frac{x}{4x^4 - 20x^2 - 3} dx$

ii. Evaluate: $\int_1^3 \log dx$

iii. In a certain culture of bacteria, their rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, find the number of times the bacteria are increased in 12 hours.

Q.3. (A) Attempt any TWO of the following questions (3 marks each): **(6) [14]**

i. A metal wire of 36 cm length is bent to form a rectangle. Find its dimensions when its area is maximum.

ii. Evaluate: $\int \frac{2x+1}{x(x-1)(x-4)} dx$

iii. Find the area of region bounded by $y^2 = 25x$ and the line $x = 4$.

(B) Attempt any ONE of the following questions (4 marks each): **(4)**

i. Using the truth table, verify:
$\sim (p \rightarrow \sim q) \equiv p \wedge \sim (\sim q) \equiv p \wedge q$

ii. Solve the following equations by method of inversion:
$x + y + z = 1$, $x - y + z = 2$, $x + y - z = 3$

(C) Attempt any ONE of the following questions (Activity) (4 marks each): **(4)**

i. The cost C for producing x articles is given as $C = x^3 - 16x^2 + 47x$. For what values of x the average cost is decreasing?

Solution:

Given $C = x^3 - 16x^2 + 47x$

Average cost $C_A = \frac{C}{x}$

$\therefore$ $C_A =$ ☐

Differentiating w.r.t. x, we get

$\frac{d}{dx}(C_A) =$ [ ]

We know that C_A is decreasing

if $\frac{d}{dx}(C_A)$ [ ] 0

$\therefore$ $2x - 16 < 0$

$\therefore$ $2x < 16$

$\therefore$ $x <$ [ ]

$\therefore$ Average cost is decreasing for $x \in (0, 8)$

ii. Solve the differential equation:

$y - x\frac{dy}{dx} = 0$

Solution:

Given equation is $y - x\frac{dy}{dx} = 0$

Separating the variables we get

$\frac{dx}{\square} = \frac{dy}{\square}$

Integrating we get,

$\int\frac{dx}{\square} = \int\frac{dy}{\square} + c$

$\therefore$ $\log x =$ [ ] $+ c$

$\therefore$ $\log x - \log y = \log c_1$, Where $c = \log c_1$

$\therefore$ $\log\left(\frac{x}{y}\right) = \log c_1$

$\therefore$ $\frac{x}{\square} = c_1$

Hence the required solution is $x = c_1 y$

Section – II

Q.4. (A) Select and write the correct answer of the following multiple choice type of questions (1 mark each): **[12] (6)**

i. The date on which the period of the bill expires is called _____.

(A) Legal due date (B) Grace date

(C) Nominal due date (D) Date of drawing

ii. A person insured a property of ₹ 4,00,000. The rate of premium is ₹ 35 per thousand p.a. The amount of annual premium is _____.

(A) ₹ 14,000 (B) ₹ 24,000

(C) ₹ 34,000 (D) ₹ 15,000

iii. Paasche's Price Index Number is given by _____.

(A) $\frac{\sum p_0q_0}{\sum p_1q_0} \times 100$ (B) $\frac{\sum p_0q_1}{\sum p_1q_1} \times 100$

(C) $\frac{\sum p_1q_0}{\sum p_0q_0} \times 100$ (D) $\frac{\sum p_1q_1}{\sum p_0q_1} \times 100$

iv. If jobs I, II, III have processing times as 8, 6, 5 on machine M_1 and 8, 3, 4 on machine M_2 in the order M_1-M_2. Then the optimal sequence is _____.

(A) I II III (B) I III II

(C) II I III (D) III II I

v. If E(X) = 4 and X follows Poisson's distribution then V(X) = _____.

(A) 2 (B) – 2

(C) 4 (D) – 4

vi. Three coins are tossed simultaneously. X is the number of heads. Then the expected value of X is _____.

(A) 1 (B) 1.5

(C) 1.9 (D) 1.017

(B) State whether the following statements are true or false (1 mark each) : **(3)**

i. In the regression of Y on X, X is the independent variable and Y is the dependent variable.

ii. The region represented by the inequalities $x \leq 0, y \leq 0$ lies in the first quadrant.

iii. In an assignment problem, if the number of columns are greater than number of rows, then a dummy column is added.

(C) Fill in the following blanks (1 mark each): **(3)**

i. If an agent charges 12% commission on the sales of ₹ 48,000 then his total commission is ₹ _____.

ii. The optimal value of the objective function is attained at the _____ points of feasible region.

iii. Given p.d.f. of a continuous random variable X is,

$$f(x) = \frac{x}{8}, \text{ for } 0 < x < 4$$

$$= 0, \text{ otherwise}$$

then $P(1 < x < 2)$ = _____.

Q.5. (A) Attempt any TWO of the following questions (3 marks each): **(6) [14]**

i. Find the rate of interest compounded annually if an immediate annuity of ₹ 20,000 per year amounts to ₹ 41,000 in 2 years.

ii. Find the Value Index Number using Simple Aggregate Method for the following data:

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	30	22	40	18
B	40	16	60	12
C	10	38	15	24
D	50	12	60	16
E	20	28	25	36

iii. Five jobs must pass through a lathe and a surface grinder, in that order. The processing times in hours are shown below. Determine the optimal sequence of the jobs. Also find the total elapsed time:

Jobs	I	II	III	IV	V
Lathe	4	1	5	2	5
Surface grinder	3	2	4	3	6

(B) Attempt any TWO of the following questions (4 marks each) : **(8)**

i. A bill was drawn on 14th April for ₹ 7,000 and was discounted on 6th July at 5% p.a. The banker paid ₹ 6,930 for the bills. Find the period of the bill.

ii. The following table gives the production of steel (in millions of tons) for years 1976 to 1986:

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Production	0	4	4	2	6	8	5	9	4	10	10

Fit a trend line to the above data by the method of least squares.

iii. Solve the following L.P.P by graphical method:

Maximize: $z = 7x + 11y$

Subject to: $3x + 4y \leq 24$,

$5x + 3y \leq 30$,

$x \geq 0, y \geq 0$

Q.6. (A) Attempt any TWO of the following questions (3 marks each) : **(6) [14]**

i. For a bivariate data, $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = 1.2$, $b_{xy} = -0.3$

Find: (a) Correlation coefficient between x and y.

(b) Estimate y for $x = 50$.

ii. Given that $\sum p_0q_0 = 220$, $\sum p_0q_1 = 380$, $\sum p_1q_1 = 350$ and Marshall-Edgeworth's Price Index Number is 150, find Laspeyre's Price Index Number.

iii. The following data gives the production of bleaching powder (in '000 tons) for the years 1962 to 1972:

Year	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
Production	0	0	1	1	4	2	4	9	7	10	8

Obtain the trend values for the above data using 5 yearly moving averages.

(B) Attempt any ONE of the following questions (4 marks each) : **(4)**

i. Four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost matrix is given below.

Machines	Places				
	A	B	C	D	E
M_1	4	6	10	5	6
M_2	7	4	-	5	4
M_3	-	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

ii. There are 10% defective items in a large bulk of items. What is the probability that a sample of 4 items will include not more than one defective item?

(C) Attempt any ONE of the following questions (Activity) (4 marks each) : **(4)**

i. The equations of the two regression lines are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. Obtain the correlation coefficient between x and y.

Solution:

To find correlation coefficient, we have to find the regression coefficients b_{yx} and b_{xy}.

Let $3x + 2y = 26$ be equation of the line of regression of y on x.

This gives $y =$ $\square$ $x + 13$

$\therefore$ $b_{yx} = -\frac{3}{2}$

Now consider $6x + y = 31$ as equation of the line of regression of x on y.

This can be written as $x =$ $\square$ $y + \frac{31}{6}$

$\therefore$ $b_{xy} = -\frac{1}{6}$

Now $r^2 =$ $\square$ $= 0.25$

As both b_{yx} and b_{xy} are negative,

$\therefore$ $r =$ $\square$

ii. The probability distribution of X is as follows:

x	0	1	2	3	4
$P(X = x)$	0.1	k	2k	2k	k

Find: (a) k
(b) $P(X < 2)$
(c) $P(1 \leq X < 4)$
(d) $F(2)$

Solution:

The table gives a probability distribution.

$\therefore \quad \sum p_i = 1$

$\therefore \quad 0.1 + k + 2k + 2k + k = 1$

(a) $k =$ ☐

(b) $P(X < 2) = P(X = 0) + P(X = 1)$
$=$ ☐

(c) $P(1 \leq X < 4) = P(1) + P(2) + P(3)$
$=$ ☐

(d) $F(2) = P(X \leq 2)$
$= P(0) + P(1) + P(2)$
$=$ ☐